EUROPEAN CENTRAL BANK
EUROSYSTEM

Working Paper Series

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Understanding inflation: insights from the term structure of inflation risks

No 3294 / October 2026

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Abstract

This paper investigates the information content of the term structure of inflation risks and its usefulness for understanding inflation dynamics. Using prices of traded zero-coupon inflation caps and floors, we develop a robust non-parametric methodology, combined with a Student's t-copula, to estimate spot and forward risk-neutral densities at short-, medium-, and long-term horizons, i.e., long-horizon risks are identified from liquid instruments alone, at daily frequency, without forward-starting contracts. Focusing on the euro area over 2009-2026, we show that the term structure of inflation risks provides valuable information about the persistence of inflation shocks and the degree to which changes in the inflation outlook become embedded at longer horizons. We also show that there is marked heterogeneity in the macroeconomic and financial conditions associated to inflation risks across horizons: short- and medium-term risks are mainly associated with current inflation, confidence indicators, commodity prices, and near-term macroeconomic risks, whereas long-term risks are more strongly related to monetary and financial conditions. These findings highlight the importance of analysing the entire term structure of inflation risks rather than relying on a single maturity.

JEL codes: G13, E31, E44
Keywords: inflation compensation, option prices, risk-neutral densities, inflation risks, financial conditions

Non-Technical Summary

Inflation expectations play a fundamental role in macroeconomics and finance: they are crucial for understanding inflation dynamics, for pricing of financial assets and for conducting monetary policy. It is therefore not surprising that their behaviour is widely monitored by researchers, market participants and central banks. Most indicators of inflation expectations, either from surveys or calculated from financial instruments often only provide information about a central scenario for future inflation. Yet, the inflation outlook is surrounded by significant uncertainty. Quantitative information on the perceived probability of different inflation outcomes can inform investment and monetary policy decisions and provide important insights for a better understanding of inflation expectations. Moreover, since the Global Financial Crisis (GFC), concerns about inflation have evolved from unprecedented deflation fears in many countries over 2015-16 following the sharp decline in commodity prices and again in 2020 following the outbreak of COVID-19, to renewed inflationary concerns triggered by supply bottlenecks along the ongoing recovery from the pandemic. These inflation developments underscore the value of any additional information about the inflation outlook at different horizons.

The goal of this paper is to assess the information content of the term structure of inflation risks. To obtain quantitative evidence on inflation risks, we use prices of inflation options, a financial instrument that through caps and floors with strikes linked to the future inflation rates offers investors the possibility of taking positions on expected inflation outcomes, in a similar fashion to interest rates or FX options, for example. Inflation options are still a relatively recent addition to the universe of inflation-linked derivatives and they come with certain limitations, such as the availability of strikes and liquidity over time. From a methodological point of view, we introduce a novel and effective approach to estimating the so-called Risk-Neutral Densities (RNDs). To estimate reliable inflation RNDs, we draw on the experience gained in other option markets, and incorporate some characteristics that help achieve substantial robustness in the estimation of RNDs. Our method is tailored not only to address the specific challenges posed by inflation options data but also to calculate implied inflation RNDs across various horizons of interest, including forward RNDs like the five-year forward in five years (5y5y), which, by reflecting the inflation outlook between five and ten years ahead, has become the standard for monitoring long-term inflation expectations.

Equipped with quantitative evidence on inflation risks at different horizons, we discuss how monitoring the term structure of inflation risks helps better understand the evolution of inflation and inflation expectations. We focus on the euro area market, which, in addition to having some of the most liquid inflation derivatives, is of particular interest given the significant swings experienced in inflation risks in recent years. By providing probabilities of inflation across a wide range of potential inflation outcomes, inflation RNDs offer crucial insights into the severity of the downward revision to the inflation outlook. Specifically, our analysis reveals that market participants revised not only mean expectations but also the balance of risks — the difference between the probability assigned to inflation above with respect to inflation turning out below the central bank target— in the light of the protracted below target inflation in the period 2013-2021. Importantly, the downward revision took place not only at short and medium-term horizons (i.e., 2-year and 5-year ahead), but also at long-horizons (i.e., 5y5y ahead). Similarly, there was a sharp revision of inflation risks towards higher outcomes at all horizons since 2022, but the risk of an entrenchment of high inflation has been contained at longer horizons.

This paper also expands the analysis of inflation RNDs by providing novel information on how inflation risks are related to economic conditions. Taking advantage of the relatively long sample available by now compared to that of earlier literature, we employ robust Bayesian selection

techniques to determine whether, when and by how much inflation risks are linked to prices and costs, economic activity and monetary and financial factors, or a combination of them. Specifically, we search for the determinants of inflation risks among 36 observable macroeconomic and financial factors belonging to 9 different categories. Our findings point to fundamental differences in the pricing of perceived inflation risks among investors at short-to-medium and longer-term horizons Headline inflation is an important determinant of inflation risks at short-to-medium horizons, but also conjunctural factors like Confidence Indicators (PMI) and commodity prices (oil), short-term Macroeconomic Risks (expected probability of high growth and high inflation one year ahead), contribute to explain inflation risks. In contrast, other types of factors like Money and Credit, Economic Activity, Stock market and International Macroeconomic factors play a very limited quantitative role. Longer-term inflation risks are also influenced by inflation developments, but financial factors (e.g. money market and credit spreads, the slope of the U.S. yield curve) play an important explanatory role, possibly shaping not only expected inflation but also inflation risk premium at long-term horizons.

1 Introduction

Inflation expectations play a central role in macroeconomics and finance. They shape inflation dynamics, affect the pricing of financial assets, and are crucial for the design and communication of monetary policy. Their analysis has benefited greatly from the development of markets for inflation-linked instruments over the past two decades. Market-based measures of inflation compensation can be constructed across a wide range of maturities and therefore provide a detailed view of the term structure of the inflation outlook. In particular, the five-year forward inflation compensation rate five years ahead (5y5y), although not directly traded, has become a standard indicator of long-term inflation compensation.[1]

Standard measures of inflation compensation, however, summarise only a central scenario for future inflation. They do not reveal how market participants assess the likelihood of alternative outcomes, the asymmetry of perceived risks, or the extent to which inflation shocks are expected to persist across horizons. These dimensions have become particularly relevant since the Global Financial Crisis. Inflation concerns shifted from persistent downside risks during the low-inflation period of the 2010s (IMF 2016) and the COVID-19 shock to pronounced upside risks during the inflation surge that began in 2021, driven initially by supply bottlenecks associated with the recovery from the pandemic and subsequently reinforced by heightened geopolitical risks (e.g., IMF 2021, 2023). This sequence of episodes highlights the need to examine not only the expected level of inflation, but also the full distribution of inflation risks at short-, medium-, and long-term horizons.

This paper studies the information content of the term structure of inflation risks in the euro area over 2009-2026. The euro area is a particularly interesting case for the analysis of inflation risks: while it shares with many other countries the post-pandemic inflation surge, it came after an unusually protracted period of below-target inflation between most of the 2010s and early 2020s and the outbreak of COVID-19. We show that evidence from the term structure of inflation risks offers important insights to understand those developments.

To obtain quantitative evidence on inflation risks at different horizons, we estimate Risk-Neutral Densities (RNDs) of inflation. Our empirical approach, detailed in Section 3, can cope well with the specific challenges posed by the traded inflation option data, and calculate implied inflation RNDs over different horizons of interest – including forward RNDs – to obtain evidence not only on the term structure of expected inflation but also on the risks surrounding it. More specifically, our non-parametric estimation approach is based on a minimal set of assumptions,[2] which provides robustness to the estimation of spot inflation RNDs, allows for asymmetries and fat tails in the inflation densities, and

[1] Market-based measures such as break-even inflation rates and inflation-linked swap rates reflect not only expected inflation, but also inflation risk premia and other market-specific components. They therefore reflect inflation expectations under the risk-neutral measure and, consequently, are more accurately interpreted as measures of inflation compensation.

[2] Existing approaches can be broadly classified as parametric, non-parametric, and semi-parametric. Parametric methods impose a specific distribution or stochastic process for the underlying asset (e.g., Melick & Thomas 1997, Bahra 1997), whereas non-parametric approaches recover the density more flexibly through spline interpolation, volatility smoothing, or kernel methods (e.g., Malz 1997, Bliss & Panigirtzoglou 2002, 2004, Fengler 2009, Malz 2014, Lai 2014). Semi-parametric approaches combine a parametric benchmark with flexible adjustments to match observed option prices (e.g., Grzelak & Oosterlee 2016, Le Floc'h & Oosterlee 2019). We adapt insights from that broader literature to the specific challenges of inflation option markets, notably the limited number of strikes, uneven liquidity, and the need for reliable tail extrapolation.

accommodates extrapolation at the tails satisfying the asymptotic no-arbitrage restriction of Lee (2004) on the tails of the implied-volatility smile. Equipped with inflation RNDs for the traded (spot) horizons, we expand the estimation to forward inflation RNDs for some specific horizons to better assess short, medium and long-term inflation risks. In particular, we link two (marginal) spot RNDs using a Student's t-copula to obtain forward RNDs for the horizons of interest that, although not traded directly in the market, have become standard metrics for inflation expectations (e.g. 5y5y for long-term expectations).

To better analyse the implications of inflation developments for monetary policy, we focus directly on the Balance of Risks (BoR henceforth) to price stability, defined as the difference between the probability mass assigned to inflation outcomes above and below the price stability objective. The BoR provides direct quantitative evidence on the severity of risks of inflation deviating from the central bank's target through its sign (upside or downside) and its magnitude. Our BoR is therefore closely related to that in Kilian & Manganelli (2008) but we use it to reflect the information of the market pricing of inflation risks instead of the preferences of the central bank.[3] We show that its evolution over time effectively summarises the information content of the whole inflation RND for price stability, and that monitoring it across time and horizons provides very useful information for monetary policymakers and investors alike.[4]

Our analysis provides two novel insights for the interpretation of inflation dynamics. First, the term structure of inflation RNDs contains important information about the persistence of changes in the inflation outlook. Section 4 shows that monitoring the balance of risks to price stability over time and across horizons (short, 2-year, medium, 5-year, and long-term, 5y5y, ahead), can help assess the extent to which shocks to inflation and inflation expectations are expected to persist over time, which is crucial for both investors and central banks. By providing probabilities of inflation across a wide range of potential inflation outcomes, inflation RNDs offer crucial insights into the severity of the revisions to the inflation outlook. Specifically, our analysis reveals that market participants revised not only mean expectations but also the balance of risks in the light of the protracted below-target inflation experienced in the euro area between 2013-2021. Moreover, the downward revision took place not only at short and medium-term horizons, but also at longer horizons. Importantly, however, the probability assigned to outright deflation (below 0%) versus low inflation outcomes (0-1.5%) suggests that while deflation risks were relatively well contained, there was a significant entrenchment of low (and below target) inflation. In contrast, there has been a sharp revision of inflation risks towards higher outcomes at all horizons since 2022, but the risk of an entrenchment of high inflation has been contained at longer horizons.

Second, risks to price stability across horizons are associated with different macroeconomic and financial factors. Taking advantage of the relatively long sample available by now, compared to that of earlier literature, we employ robust Bayesian selection techniques (George & McCulloch 1993) to determine whether, when and by how much inflation risks are linked to prices and costs, economic activity and monetary and financial factors, or a combination of them. Specifically, we search for the determinants of inflation risks

[3] Kilian & Manganelli (2008) finds that monetary policy decisions under Greenspan are better described as taking into account upside and downside risks to their objectives rather than simply responding to the conditional mean of inflation (and of the output gap).

[4] In addition, we will also use information on the probability assigned to values far from the inflation objective to further qualify the key messages from the BoR. Specifically we use lower tail risk as captured by the probability of outright deflation outcomes (i.e. below 0%) as well as the upper tail risks to better understand the evolution of inflation risks.

among 36 observable macroeconomic and financial variables belonging to nine different categories. Our findings point to fundamental differences in the pricing of perceived inflation risks among investors at short-to-medium and longer-term horizons (see Section 5). As expected, inflation and long-term inflation expectations (ECB's SPF five year ahead) are important determinants of inflation risks at all horizons. Short-to-medium horizons are also strongly influenced by cyclical factors like Confidence Indicators (PMI), commodity prices (oil and raw materials), short-term Macroeconomic Risks (expected probability of high growth one year ahead). Longer-term inflation risks are also influenced by developments in Price and Cost indicators (PPI), but financial factors (OIS spreads, Credit Spreads and the slope of the U.S. yield curve) also play an important explanatory role, possibly shaping not only expected inflation but also inflation risk premia at long-term horizons. Instead, oil and commodity prices and confidence indicators, which, as discussed above, explain a substantial part of the variation in perceived risks over the next five years, do not seem to be relevant at longer horizons. In contrast, other types of factors like Money aggregates and Credit, Economic Activity, Stock market and International macroeconomic factors play a very limited quantitative role.

This paper makes both methodological and empirical contributions to the related literature. Methodologically, we provide a unified framework for recovering spot and non-traded forward inflation densities while preserving the flexibility needed to capture non-Gaussian features of the distribution. This flexibility matters because, under any symmetric density, the sign of the balance of risks is fixed by the mean, so risks can tilt downward only when expected inflation sits below target. The configurations we document (expectations near target but probability mass tilted to one side) are then unrepresentable, and the balance of risks collapses into the point measures of inflation compensation whose insufficiency motivates this study. Recovering such densities at horizons where no instrument trades, at thresholds outside the quoted strike range, and on dates when liquidity is poorest is also what parametric and strike-based approaches in the existing literature cannot provide. Within the inflation options literature, early studies focused mainly on spot distributions for particular markets (e.g., Smith 2012, Kitsul & Wright 2013, Scharnagl & Stapf 2015, Fleckenstein et al. 2017). More recent work has extended the analysis to forward densities using multivariate Gaussian structures, Gaussian copulas, or frameworks designed to study inflation disasters (e.g., Gimeno & Ibáñez 2018, Hilscher et al. 2022, 2026). We contribute to this literature by combining non-parametric marginal densities with a Student's t-copula and by analysing the entire distribution of inflation risks across short-, medium-, and long-term horizons. This choice matters: a Gaussian copula rules out tail dependence between horizons whatever the correlation, whereas the tail dependence we estimate is substantial at most dates in our sample (Appendix F.6). Empirically, we show that the term structure of inflation risks reveals information about the persistence of shocks that no single maturity can provide. The short- and long-horizon distributions differ in more than the magnitude of their responses: over extended periods they carry opposite signs or move in opposite directions, so the long-term horizon density cannot be dismissed as an attenuated version of the short-term one (Section 4). Bayesian variable selection over 36 macroeconomic and financial indicators confirms this: the two ends of the term structure load on different information sets (Section 5). Together both contributions support a single proposition: that the term structure of inflation RNDs identifies how persistently inflation shocks are priced.

Our discussion of inflation risks in the euro area over recent years is closely related to that in Gimeno & Ibáñez (2018) and Hilscher et al. (2026). There are, however,

important differences. Hilscher et al. (2026) for instance use a different methodology for the estimation of spot and forward inflation RNDs in both the U.S. and the euro area, adjust the estimated RND probabilities (for real payoffs, risk, and horizon), and emphasise the evolution of inflation disasters as reflected in the "tail risks", in particular the evolution of the probability of deflation in the euro area at long horizons. Our analysis instead focuses on the analysis of the term structure of inflation risks, comparing the balance of risks implied by the whole inflation RND at the commonly-used horizon 5y ahead to forward RNDs reflecting short (i.e., 2y) and long-term risks (i.e., 5y5y). Finally, our findings regarding that different macroeconomic and financial factors are associated with inflation risks at different points along the term structure, also complement studies of the response of inflation tail probabilities to macroeconomic news and monetary policy announcements (e.g., Kitsul & Wright 2013, Scharnagl & Stapf 2015, Galati et al. 2018).

As in Hilscher et al. (2026), our paper also uses inflation options to characterise the distribution of future inflation and both studies argue that point measures of inflation compensation are insufficient. The two papers are complementary and differ in the method and the data required. In Hilscher et al. (2026) probabilities over eight inflation bins defined by strikes are recovered and forward probabilities are constructed by combining forward-starting one-year options with a first-order Markov model of inflation dynamics whose transition matrix is restricted by a disaster process. In contrast, we non-parametrically estimate a continuous density at each traded maturity and obtain the 5y5y forward density by coupling the five- and ten-year distributions with a Student's t-copula.

Consequently, there are the following important differences between the two works. First, our estimation of forward RNDs (such as 5y5y) requires only the zero-coupon options at the two bracketing maturities (i.e., it rests on data from the liquid market) and does not use forward-starting contracts, which are traded in much less liquid markets. Second, because our estimates are continuous rather than binned, we evaluate probabilities at thresholds that need not coincide with traded strikes, including the 1.5% boundary used in our analysis of risks to price stability, and we obtain the moments of the distribution directly. Third, we estimate at daily rather than monthly frequency and we report end-of-month observations throughout because the episodes we analyse unfold over months rather than days. At the same time, daily estimates make the framework usable for event studies of the response of the whole term structure of inflation risks to policy announcements and data releases, which is an interesting application that we leave for future work. On the other hand, Hilscher et al. (2026) adjust risk-neutral probabilities to physical ones using cross-country evidence on marginal utility during inflation disasters, whereas we work under the risk-neutral measure. Our primary focus is on the entire distribution at short-, medium-, and long-term horizons, and on the macroeconomic and financial conditions associated with it, which we examine in Section 5.

Our results show that the term structure of inflation risks carries information that no single maturity conveys. In the first half of 2020 the risk-neutral probability of outright deflation reached almost 70% two years ahead and around 36% at five years, but remained below 7% for 5y5y ahead. At that horizon the dominant market anticipation was not deflation but the persistence of low inflation with more than 70% of the probability mass between 0% and 1.5%. The post-2021 surge produced the same asymmetry in the opposite direction. By spring 2022 almost all of the probability mass lay above 2% at the two- and five-year horizons, while the long-horizon distribution moved far less and far later. In both periods, market participants repriced the near-term outlook sharply and left the

5y5y ahead distribution comparatively anchored. That distinction is invisible in point measures of inflation compensation stressing the significance of the whole term structure of risks, rather than any single maturity, for monetary policy decisions.

2 The euro area inflation derivatives market

This section describes the inflation-linked instruments and market data used in the analysis. Section 2.1 introduces the euro area inflation swap and option markets and summarises the dataset. Section 2.2 then presents the payoffs and risk-neutral valuation of zero-coupon inflation swaps, caps, and floors. These definitions establish the mapping between inflation options and standard European options that underlies the recovery of inflation risk-neutral densities in Section 3.

2.1 Inflation-linked instruments and data

Inflation-linked markets provide direct information about market participants' valuation of future inflation. Zero-coupon inflation-indexed swaps (ZCIIS) measure inflation compensation over different maturities, while inflation caps and floors provide information about the distribution of risks surrounding that compensation. Together, these instruments allow us to study not only the central inflation outlook but also the term structure of the associated risks.

Our analysis focuses on the euro area inflation derivatives market, the most developed market for inflation-linked swaps and options. It offers a broad range of zero-coupon inflation swaps, caps, and floors across maturities and strikes, as well as a longer and more liquid option-price history than comparable markets. These features make the euro area particularly suitable for recovering and analysing the full term structure of inflation risks.

We use daily zero-coupon inflation swap rates and zero-coupon inflation cap and floor prices for maturities ranging from one to ten years, together with overnight indexed swap rates for discounting. The data are obtained from Bloomberg and cover the period from October 2009 to February 2026. The reference index for the inflation contracts is the euro area Harmonised Index of Consumer Prices excluding tobacco (HICPxT).

Inflation option quotes are available only for a finite set of strikes, and their liquidity and reliability vary across maturities, strikes, and time. We therefore apply a series of data-quality filters before estimating the risk-neutral densities. In particular, we remove quotes that violate put–call parity or imply non-convex option-price curves. Whenever both cap and floor quotes are available at the same strike, we retain the out-of-the-money contract. These characteristics of the data motivate the interpolation, smoothing, and extrapolation procedure introduced in Section 3.1.

2.2 Zero-coupon inflation swaps and options

This subsection recalls the definitions and risk-neutral valuation of the inflation-linked instruments used in the paper. Further details on the pricing of zero-coupon inflation swaps, caps, and floors are provided in Appendix A. For detailed treatments of inflation-linked markets and their pricing, see Deacon et al. (2004), Canty & Heider (2012), and Brigo & Mercurio (2007).

Let $I(t)$ denote the relevant consumer price index. The annualised inflation rate between dates t and T, denoted by $\pi(t,T)$, is defined by

$$\frac{I(T)}{I(t)} = (1+\pi(t,T))^{T-t}. \tag{1}$$

Accordingly, an XyZy inflation rate refers to the annualised inflation rate over the X-year period beginning Z years ahead. For example, the 5y5y inflation rate is $\pi(5,10)$.

A zero-coupon inflation-indexed swap with maturity T exchanges the realised cumulative inflation payoff

$$N\left[(1+\pi(0,T))^T - 1\right]$$

against the fixed payoff

$$N\left[(1+k)^T - 1\right],$$

where N is the notional and k is the fixed swap rate. The swap rate is set such that the contract has zero value at inception. Using the change-of-numeraire argument of Jamshidian (1989), the fair swap rate satisfies

$$(1+k)^T = \mathbb{E}_{\mathbb{Q}^T}\left[(1+\pi(0,T))^T\right], \tag{2}$$

where $\mathbb{Q}^T$ is the T-forward measure associated with the nominal zero-coupon numeraire for maturity T (see also Brigo & Mercurio 2007).

Let $P_n(t,T)$ denote the time-t nominal discount factor for maturity T, corresponding to the value at time t of receiving one unit of currency at time T.

A zero-coupon inflation cap with strike k and maturity T pays

$$\left[(1+\pi(0,T))^T - (1+k)^T\right]_+$$

at maturity. Its time-t price is

$$C(t;T,k) = P_n(t,T)\mathbb{E}_{\mathbb{Q}^T}\left[\left((1+\pi(0,T))^T - (1+k)^T\right)_+ \middle| \mathcal{F}_t\right]. \tag{3}$$

Similarly, a zero-coupon inflation floor pays

$$\left[(1+k)^T - (1+\pi(0,T))^T\right]_+$$

at maturity, and its time-t price is

$$P(t;T,k) = P_n(t,T)\mathbb{E}_{\mathbb{Q}^T}\left[\left((1+k)^T - (1+\pi(0,T))^T\right)_+ \middle| \mathcal{F}_t\right]. \tag{4}$$

Caps and floors with the same strike and maturity satisfy the put–call parity relation

$$C(t;T,k) - P(t;T,k) = P_n(t,T)\left\{\mathbb{E}_{\mathbb{Q}^T}\left[(1+\pi(0,T))^T \middle| \mathcal{F}_t\right] - (1+k)^T\right\}. \tag{5}$$

At inception, the expectation in Equation (5) can be expressed using the prevailing inflation swap rate through Equation (2). This relation allows floor prices to be converted into call-equivalent prices, as described in Section 3.1.

Equations (3) and (4) show that zero-coupon inflation caps and floors can be interpreted, respectively, as European call and put options written on the cumulative inflation index ratio

$$\frac{I(T)}{I(0)} = (1 + \pi(0,T))^T ,$$

with strike

$$K = (1 + k)^T .$$

This correspondence provides the basis for recovering the risk-neutral density of cumulative inflation from zero-coupon inflation option prices using the approach of Breeden & Litzenberger (1978).

3 Estimating inflation risk-neutral densities

This section presents the methodology used to recover the term structure of inflation risk-neutral densities (RNDs) from zero-coupon inflation option prices. We proceed in two steps. First, we estimate spot RNDs at traded maturities using a flexible non-parametric procedure that smooths and extrapolates the implied-volatility smile satisfying the asymptotic tail-growth restriction of Lee (2004). Second, we construct forward RNDs for non-traded horizons by combining the estimated marginal distributions through a Student's t-copula. This allows us to recover inflation densities at short-, medium-, and long-term horizons within a unified framework.

The methodology addresses three features of inflation option markets. Option prices are available only for a finite and relatively small set of strikes, the liquidity and reliability of quotes vary across strikes and over time, and recovering the tails of the distribution requires extrapolation beyond the range of observed prices. Our procedure accommodates these features while allowing the estimated densities to display asymmetry and excess kurtosis when supported by the data. Further implementation details, a worked empirical example, and the robustness exercises are reported in Appendices B–F.

3.1 Estimating zero-coupon inflation RNDs

As shown in Section 2.2, zero-coupon inflation caps and floors can be represented as European call and put options written on the cumulative inflation index ratio $I(T)/I(0)$, with transformed strike $K = (1 + k)^T$. Using put–call parity, floor quotes are converted into call-equivalent prices, and whenever both a cap and a floor are available at the same strike, we retain the out-of-the-money contract.

Let $C(t; T, K)$ denote the resulting time-t call price for maturity T and strike K. When option prices are observed over a continuum of strikes, the risk-neutral density of the underlying can be recovered from the second derivative of the option-price function with respect to the strike (Breeden & Litzenberger 1978):

$$\frac{\partial^2 C(t; T, K)}{\partial K^2} = P_n(t, T) q_S^T(T, K), \tag{6}$$

where $P_n(t, T)$ is the nominal discount factor and $q_S^T(T, K)$ denotes the density of the cumulative inflation index ratio under the T-forward measure.

In practice, option prices are observed for only a finite set of strikes. Interpolating prices directly and differentiating the resulting curve twice can amplify small pricing

irregularities, leading to unstable or locally non-convex estimates. We therefore express the observed prices in terms of Black implied volatilities and smooth the implied-volatility smile instead. Smoothing in volatility space substantially reduces this problem. For given forward and discount factors, implied volatility is in one-to-one correspondence with the option price, so this transformation preserves the information contained in the observed quotes while providing a more suitable object for interpolation and extrapolation (Shimko 1993).

Let $\sigma_{\text{imp}}(K)$ denote the implied volatility associated with strike K. To facilitate smooth extrapolation while satisfying the asymptotic growth restriction of Lee (2004), we transform implied volatility according to

$$\widetilde{\sigma}(K) = \exp\left\{\sigma^2_{\text{imp}}(K)\right\}. \tag{7}$$

This transformation follows Maurer et al. (2019). Appendix B shows that polynomially bounded extrapolations of $\widetilde{\sigma}(K)$ satisfy the required growth condition.

We fit a natural cubic smoothing spline $g(K)$ to the transformed implied volatilities. The spline minimises

$$(1-\alpha)\sum_{i=1}^{n}\left[\widetilde{\sigma}_i - g(K_i)\right]^2 + \alpha\int_{K_1}^{K_n}\left[g''(x)\right]^2 dx, \tag{8}$$

where $\alpha \in [0,1]$ governs the trade-off between goodness of fit and smoothness (De Boor 1978). Cubic smoothing splines have been widely used to recover implied-volatility smiles (e.g., Bliss & Panigirtzoglou 2002, 2004, Fengler 2009). In the benchmark specification, we set $\alpha = 0.5$; the sensitivity of the estimated probabilities to alternative degrees of smoothing is examined in Appendix E.

The spline is extrapolated linearly beyond the observed strike range using its endpoint slopes. Since the extrapolation is applied to the transformed implied volatility, the resulting implied-volatility function satisfies the required tail-growth restriction and avoids artificial truncation or discontinuities in the implied density.

Finally, the fitted implied-volatility function and its analytical derivatives are substituted into Equation (6). This yields the RND of the cumulative inflation index ratio and, through a change of variables, the RND of the annualised inflation rate $\pi(0,T)$. The analytical expression is reported in Appendix C. Using the analytical derivatives of the spline avoids the numerical instability associated with second-differencing a fitted option-price curve.

The procedure is applied separately at each traded maturity and observation date. A worked example using five-year euro area inflation options is provided in Appendix D.

3.2 Modelling forward RNDs using Student's *t*-copula

Zero-coupon inflation options provide risk-neutral densities for cumulative inflation between the current date and a traded maturity. They do not, however, directly reveal the distribution of inflation over a future interval. In particular, no option is directly traded on the five-year inflation rate five years ahead. Recovering the 5y5y RND therefore requires combining the marginal distributions of cumulative inflation at five- and ten-year maturities with an assumption about their dependence.[5]

[5]The construction of non-traded forward RNDs necessarily requires an assumption about the dependence between marginal distributions. Hilscher et al. (2026) draw on the rare-disasters literature to model

Let $0 < T_1 < T_2$. The annualised inflation rate over the interval $[T_1, T_2]$ satisfies

$$(1 + \pi(T_1, T_2))^{T_2 - T_1} = \frac{I(T_2)}{I(T_1)}. \tag{9}$$

Since

$$\frac{I(T_2)}{I(T_1)} = \frac{I(T_2)/I(0)}{I(T_1)/I(0)}, \tag{10}$$

the forward inflation density can be obtained from the joint distribution of the two cumulative inflation ratios

$$\frac{I(T_1)}{I(0)} \qquad \text{and} \qquad \frac{I(T_2)}{I(0)}.$$

Their marginal densities are recovered from zero-coupon inflation option prices using the procedure described in Section 3.1.

We model the dependence between these two marginal distributions using a Student's t-copula. Relative to a Gaussian copula, the Student's t-copula allows for non-zero symmetric tail dependence and can therefore accommodate joint extreme movements in inflation risks across maturities. Appendix F.1 defines the coefficient of tail dependence and gives its closed form (note that a Gaussian copula rules tail dependence out whatever the value of the correlation parameter).

Let f_{T_1} and f_{T_2} denote the marginal densities, and let F_{T_1} and F_{T_2} be their corresponding cumulative distribution functions. The joint density is specified as

$$f_{T_1,T_2}(t_1, t_2) = c_{\rho,\nu}\left(F_{T_1}(t_1), F_{T_2}(t_2)\right) f_{T_1}(t_1) f_{T_2}(t_2), \tag{11}$$

where $c_{\rho,\nu}$ is the density of a bivariate Student's t-copula, ρ governs dependence, and ν controls tail thickness. The formal definition of the copula is reported in Appendix F.

Following Gimeno & Ibáñez (2018), we assume deterministic interest rates. Under this assumption, the forward measures associated with different maturities coincide with a common risk-neutral measure $\mathbb{Q}$. Hence, the zero-coupon inflation RNDs at maturities T_1 and T_2 can be combined consistently under the same pricing measure.

Given the joint density in Equation (11), the density of the cumulative forward inflation ratio

$$X = \frac{I(T_2)}{I(T_1)} \tag{12}$$

is,

$$q_{T_1,T_2}(x) = \int_{\mathbb{R}} |t_1| f_{T_1,T_2}(t_1, x t_1)\, dt_1, \tag{13}$$

following the standard density transformation for the quotient of two random variables (Curtiss 1941). Since price indices are positive, the integral can equivalently be evaluated over the positive real line. The RND of the annualised forward inflation rate $\pi(T_1, T_2)$ is then obtained through the change of variables implied by Equation (9). The corresponding expression is reported in Appendix F.

Let $k_T(t)$ denote the observed T-year zero-coupon inflation swap rate at date t, and define

$$K_T(t) = \left(1 + k_T(t)\right)^T. \tag{14}$$

long-horizon transition probabilities using euro area year-on-year inflation options at five-, seven-, and ten-year maturities, while Hilscher et al. (2022) construct forward RNDs for the United States using year-on-year options and a Gaussian copula.

Under the deterministic-interest-rate assumption, $K_T(t)$ is proportional to the forward CPI associated with maturity T and therefore embeds the current risk-neutral valuation of $I(T)$. We define its daily change as

$$\Delta K_T(t) = K_T(t) - K_T(t-1). \tag{15}$$

At each date t, the copula parameters are estimated over a rolling window of n trading days using the paired observations

$$\{(\Delta K_{T_1}(s), \Delta K_{T_2}(s))\}_{s=t-n+1}^{t}. \tag{16}$$

These changes capture the contemporaneous dependence in the risk-neutral repricing of cumulative inflation across maturities. The benchmark specification uses a window of 100 trading days and only information available up to each estimation date. The sensitivity of the results to alternative copulas and window lengths is examined in Appendix E.

3.3 Robustness tests

Appendix E reports three sets of robustness exercises. First, using synthetic option prices generated by a SABR-type inflation model, we show that the interpolation and extrapolation procedure recovers the underlying probabilities with high accuracy, with somewhat larger errors only in the most extreme tails. Second, the Student's t-copula provides a better fit than the Gaussian copula. Although the grouped Student's t-copula can achieve a marginally better fit when estimation is successful, it is less numerically stable. The standard Student's t-copula therefore offers the best balance between flexibility, fit, and convergence. Third, the resulting forward RNDs are robust to using rolling windows of 45, 100, or 150 trading days.

4 Assessing risks to price stability: insights from the term structure of inflation RNDs

This section examines what option-implied inflation RNDs reveal about risks to price stability. Unlike inflation swap rates and other point measures of inflation compensation, inflation options offer information about the distribution of market valuations across alternative future inflation outcomes. A growing literature has used this information to quantify uncertainty, asymmetry and tail risks in inflation markets (Kitsul & Wright 2013, Scharnagl & Stapf 2015, Hilscher et al. 2026). We contribute to this literature by combining non-parametric marginal RNDs with a Student's t-copula and by analysing the entire distribution of inflation risks across short-, medium-, and long-term horizons. A Gaussian copula rules out tail dependence between maturities whatever the underlying correlation, while the Student's t-copula allows for it and the corresponding governing parameter is estimated at each date.

Our analysis rests on two elements. First, motivated by the risk-management approach to monetary policy (Kilian & Manganelli 2007, 2008, Gimeno & Ibáñez 2018), we focus on the balance of risks (BoR) around the price stability benchmark. The BoR compresses the information contained in the entire RND into a single, directly interpretable statistic by comparing the total probability mass assigned to inflation outcomes above and below that objective. Unlike measures based on particular tails or selected moments, it depends

on the whole distribution: changes in its location, dispersion, asymmetry or tail thickness affect the BoR whenever they alter the allocation of probability mass around the price stability objective. The measure therefore captures both the direction and the magnitude of the imbalance in risks to price stability.

Second, we jointly analyse spot and forward inflation RNDs at three horizons: the two-year RND, capturing short-term risks; the five-year RND, capturing risks over the medium term; and the five-year forward RND five years ahead (5y5y), capturing the true long-term outlook for inflation risks. The 5y5y RND captures inflation risks between five and ten years ahead, the horizon conventionally used by central banks and international institutions to assess long-term inflation compensation and the anchoring of inflation expectations (ECB 2018, IMF 2023, Kashkari 2022). Comparing shifts in the three inflation RNDs allows to distinguish transitory or cyclical reactions in short or medium-term RNDs from more persistent shifts also priced in at longer horizons. The term structure of inflation RNDs therefore provides information about whether inflation shocks are perceived as transitory or sufficiently persistent to threaten price stability over the long term.

All the probabilities analysed in this section are calculated under the risk-neutral measure, $\mathbb{Q}$, because they are recovered from traded inflation options. They should therefore not be interpreted as physical probabilities or direct forecasts of inflation outcomes. Risk-neutral densities reweight physical probabilities according to the market prices of the corresponding states and thus reflect both beliefs about future inflation and the pricing of inflation risk (Kitsul & Wright 2013). Separating these two components would require additional identifying assumptions about the physical distribution and the inflation risk premia. We therefore interpret the RNDs throughout as market-implied risk-neutral assessments of inflation risks.

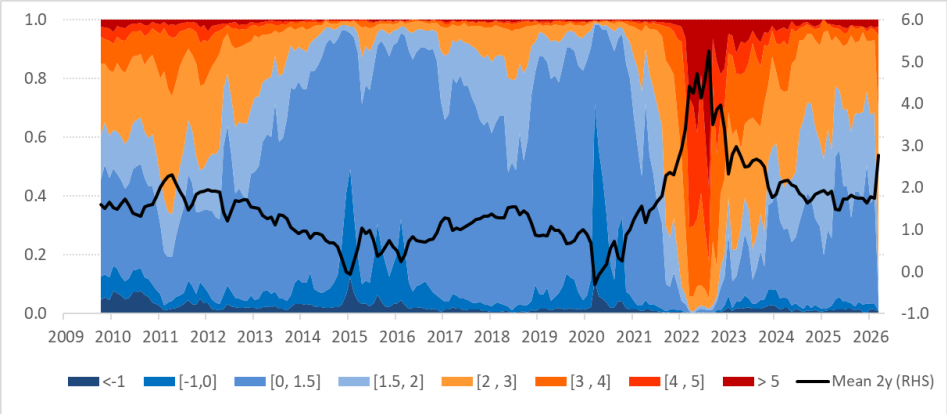
Our analysis focuses on the euro area between October 2009 and February 2026. This period provides a particularly informative setting, encompassing the sovereign debt crisis and the subsequent prolonged period of below-target inflation, the COVID-19 shock, and the sharp post-pandemic rise in inflation. We first describe the evolution of the estimated distributions across the short-, medium- and true long-term horizons and then use the BoR to capture the direction, dispersion, asymmetry and tail thickness of inflation risks. Although the RNDs are estimated at a daily frequency, the figures report end-of-month observations in order to emphasise the main shifts over time while preserving a well-defined market information set at each observation.[6]

4.1 The term structure of inflation RNDs

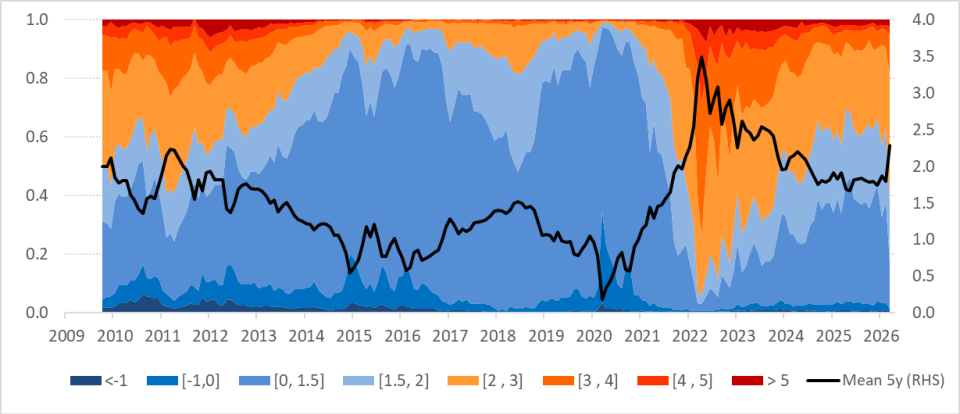
Figure 1 reports the risk-neutral probability mass assigned to a set of economically relevant inflation ranges over short, medium and long horizons. Intervals comprise outright deflation, defined as annualised inflation below 0%; low inflation between 0% and 1.5%; inflation close to but below the 2% price-stability benchmark, between 1.5% and 2%; moderately above-benchmark inflation between 2% and 3%; and progressively higher inflation outcomes in the intervals from 3% to 4%, from 4% to 5%, and above 5%. The solid black line reports the mean of the corresponding RND and therefore provides a measure of the central market-implied inflation outlook against which the distribution of risks can be assessed.

Three broad features emerge from the figures. First, both the central outlook and the

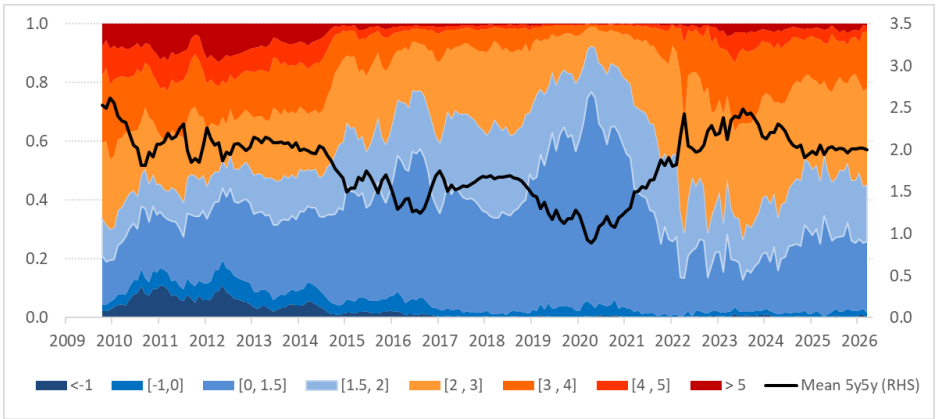
[6] A monthly average would combine RNDs conditioned on different daily information sets and would not generally correspond to the market-implied distribution prevailing on any particular date.



(a) Short-term horizon (2y RND)



(b) Medium-term horizon (5y RND)



(c) Long-term horizon (5y5y RND)

Figure 1: **Expected mean and probabilities of inflation at different horizons.** Note: The charts depict the probability assigned to future inflation turning out within different value ranges together with the mean expected value (black line) at short, medium and long-term horizons according to the risk-neutral densities estimated in this paper.

distribution of inflation risks become progressively more stable as the horizon lengthens. The two-year RND reacts strongly to current inflation developments and other transitory influences, whereas the 5y5y RND displays substantially smaller and more gradual shifts. This cross-horizon pattern is consistent with long-term inflation pricing being more firmly anchored than the short-term outlook.

Second, the figures reveal a persistent leftward shift in the term structure of inflation risks during the period of low inflation that extended throughout much of the 2010s. The movement was initially most visible at the two- and five-year horizons, where probability mass became increasingly concentrated in low and below-target outcomes. Over time, however, the on-going deterioration in the inflation outlook also became apparent in the 5y5y RND. The fact that the shift eventually extended to the distribution of inflation between five and ten years ahead indicates that market concerns were not confined to temporarily weak inflation, but increasingly encompassed the possibility of persistently low inflation at long-term horizons.

Third, the post-pandemic increase in inflation produced a sharp reversal in the distribution of risks. From 2021 onwards, probability mass moved rapidly towards above-benchmark outcomes, particularly at the two-year horizon. The repricing subsequently extended to the five-year RND, whereas the adjustment in the 5y5y distribution was considerably more moderate. Thus, although the inflation surge affected the entire term structure, option prices continued to differentiate between pronounced near-term inflation pressures and a more contained reassessment of risks between five and ten years ahead.

4.2 The balance of risks to price stability

A central concern for monetary policy is not only the most likely inflation outcome, but also the balance of risks surrounding the price-stability objective. Inflation RNDs provide detailed information about the probability assigned to alternative outcomes. Probabilities of outright deflation or very high inflation, for example, are informative about particular scenarios in the lower and upper tails of the distribution (Hilscher et al. 2026). However, they capture only selected parts of the RND and may therefore overlook broader reallocations of probability mass across inflation outcomes.

Centred moments such as the standard deviation, skewness and kurtosis provide complementary information about the dispersion, asymmetry and tail thickness of the RND. Their interpretation from a monetary-policy perspective is nevertheless less direct because they are defined relative to the endogenous mean of the distribution rather than to the central bank's price-stability objective. For monitoring risks to price stability, we will instead focus on a metric defined directly relative to the policy objective.

We employ the BoR to summarise the allocation of probability mass around the central bank's price-stability objective. Formally, let $q_t^{\mathbb{Q}}(x)$ and $F_t^{\mathbb{Q}}(x)$ denote, respectively, the risk-neutral density and cumulative distribution function of inflation at date t for the horizon considered, π_t denote the corresponding inflation rate, and π^* denote the benchmark inflation level for price stability. For a continuous RND, the BoR is defined as

$$\mathrm{BoR}_t = \mathbb{Q}(\pi_t > \pi^*) - \mathbb{Q}(\pi_t < \pi^*) = \int_{\pi^*}^{\infty} q_t^{\mathbb{Q}}(x)\,dx - \int_{-\infty}^{\pi^*} q_t^{\mathbb{Q}}(x)\,dx = 1 - 2F_t^{\mathbb{Q}}(\pi^*). \quad (17)$$

The BoR maps the entire inflation RND into a single, directly interpretable statistic. Since it aggregates all probability mass above and below the benchmark, it incorporates

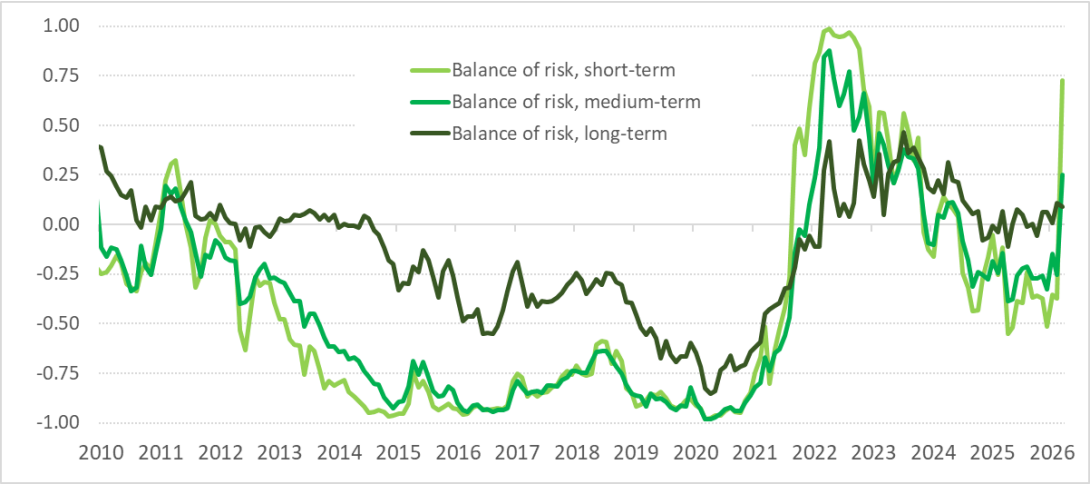


Figure 2: **The balance of risks at different horizons.**
Note: The figure reports the difference between the risk-neutral probability assigned to annualised inflation outcomes above 2% and that assigned to outcomes below 2%. The BoR is calculated from the two-year, five-year and 5y5y inflation RNDs using end-of-month observations over the period 2009M10-2026M2.

the combined effects of changes in the location, dispersion, asymmetry, tail thickness and other features of the distribution whenever they alter the allocation of probability around π^*. It therefore provides a whole-distribution measure rather than one based on a particular tail or selected moment.

The BoR ranges between -1 and 1. A positive value indicates that the RND assigns more probability mass to inflation outcomes above the benchmark than below it, signalling predominantly upside risks to price stability. A negative value indicates predominantly downside risks. Its absolute magnitude measures the degree of directional imbalance: values close to one in absolute terms imply that most of the probability mass lies on one side of the benchmark, whereas a value of zero indicates equal probability mass on either side. A zero BoR denotes balanced risks, however, rather than an absence of uncertainty or tail risk; a distribution may be highly dispersed or fat-tailed while assigning equal probability mass to both sides of the benchmark. For comparability over time, we set $\pi^* = 2\%$ throughout the sample.[7]

Figure 2 reports the BoR for the two-year, five-year and 5y5y RNDs. The three series measure the directional imbalance of inflation risks at the short-term, medium-term and true long-term horizons, respectively.

The figure reveals substantial common variation in the direction of inflation risks across horizons, but also economically relevant differences in their timing and magnitude. Short- and medium-term risks generally display larger and more rapid movements, whereas the 5y5y BoR is less volatile and tends to respond more gradually. These cross-horizon differences provide information about whether changes in inflation risks are concentrated in the nearer-term outlook or extend to the true long-term horizon. The main historical episodes underlying these movements are examined in Section 4.3.

[7]Until the 2021 Monetary Policy Strategy Review, the ECB aimed to maintain inflation below but close to 2% over the medium term. The Strategy Assessment of July 2021 has defined price stability as a symmetric 2% inflation target over the medium term (European Central Bank 2003, 2021). Using 2% as a constant benchmark provides a common numerical reference for comparing risks over the full sample.

4.3 Challenges to price stability

The term structure of inflation RNDs identifies three major episodes of risks to price stability in the euro area over the sample period. The first was the progressive entrenchment of low and below-benchmark inflation during the 2010s. The second was the further deterioration in the inflation outlook following the outbreak of COVID-19 in 2020. Both episodes were characterised by predominantly negative BoR values, although their distributional patterns and propagation across horizons differed substantially. The third episode was the post-pandemic inflation surge, which generated a rapid shift towards positive BoR values from early 2021.

The comparison across episodes illustrates the two main advantages of the framework developed in this paper. First, the BoR captures broad reallocations of probability mass that may not be visible from the analysis of extreme tail events alone. Second, its term structure reveals whether a deterioration in the inflation outlook is concentrated at shorter horizons or extends to the true long-term distribution.

4.3.1 From cyclical downside risks to the entrenchment of low inflation

The term structure of inflation RNDs reveals an important distinction between the downside risks observed during the Global Financial Crisis and the euro area sovereign debt crisis, on the one hand, and those that emerged from 2014 onwards, on the other. In 2009, and again during the most acute phase of the sovereign debt crisis, the deterioration was concentrated primarily at shorter horizons. These episodes involved severe contemporaneous contractions in economic activity and therefore increased the probability of weak or negative inflation over the following years. The decline in the short-horizon BoR, including the two-year measure reported here, is consistent with markets pricing the near-term disinflationary consequences of those crises.

The relative stability of the 5y5y BoR during the Global Financial Crisis and the sovereign debt crisis was particularly informative. Despite the weakness of the near-term outlook, the long-term distribution remained broadly centred around the price stability benchmark. Markets therefore initially appeared to interpret the disinflationary effects of the Global Financial Crisis and the sovereign debt crisis as predominantly cyclical, rather than as a permanent change in the euro area's inflation process. This interpretation is consistent with the ECB's contemporaneous assessment that temporary negative inflation rates and weak activity had not materially disturbed the anchoring of medium- to long-term inflation expectations (European Central Bank 2009, 2012).

The pattern changed markedly from 2014 onwards. The two- and five-year BoR had already moved into negative territory, but the subsequent decline in the 5y5y BoR showed that the perceived risk was no longer confined to the near-term consequences of weak economic activity. Probability mass shifted towards below-benchmark outcomes even for inflation between five and ten years ahead. The simultaneous deterioration of the short-, medium- and true long-term RNDs therefore indicated a growing risk that low inflation would become entrenched.

This distinction is economically important. A negative short-term BoR may reflect the conventional response of inflation to a temporary recessionary shock. A persistently negative 5y5y BoR instead indicates that markets assign substantial probability to inflation remaining below the price-stability benchmark after the immediate effects of the shock should have dissipated. The latter represents a more fundamental challenge for monetary policy because it points to a weakening of the nominal anchor rather than merely to

temporary cyclical disinflation.

The composition of the RNDs confirms that the dominant concern during the second half of the 2010s was not outright deflation. Between 2014 and 2019, the probability of negative inflation averaged around 6% at short and medium horizons, notwithstanding temporary increases during 2015–16. By contrast, the probability assigned to inflation between 0% and 1.5% averaged approximately 70%. At the true long-term horizon, the corresponding probabilities were below 5% for deflation and approximately 39% for inflation between 0% and 1.5%. Thus, even when the probability of an extreme deflationary outcome remained relatively limited, the BoR was strongly negative because a large share of the distribution was concentrated at low positive inflation rates materially below the price-stability benchmark.

This result clarifies the relationship between our analysis and the inflation-disaster approach of Hilscher et al. (2026). Tail probabilities provide valuable information about the likelihood of extreme outcomes such as outright deflation or very high inflation. The BoR addresses a broader question by capturing reallocations of probability mass across the entire distribution relative to the policy benchmark. During the 2010s, an exclusive focus on the lower tail could therefore suggest that deflation risks were generally contained, while overlooking the more persistent policy challenge created by the concentration of probability mass in the 0–1.5% range.

The deterioration of the 5y5y BoR is also consistent with evidence of a weakening in the anchoring of euro area inflation expectations (Natoli & Sigalotti 2018, Garcia & Werner 2021). The long-term BoR remained broadly balanced during 2011–13, even after the two- and five-year measures had already turned negative. It moved decisively below zero only in the second half of 2014, around the time when both the mean of the 5y5y RND and standard measures of long-term inflation compensation fell below 2%. From 2015 onwards, the presence of negative BoR values at all three horizons indicated that low inflation was increasingly perceived as persistent rather than as the transitory consequence of a cyclical downturn.

The timing of this deterioration coincided with a major change in the ECB's monetary-policy response. Although the ECB continued to describe medium- to long-term inflation expectations as firmly anchored in early 2014, it also warned that an excessively prolonged period of low inflation could eventually affect expectations. In June 2014, the Governing Council reduced the deposit facility rate below zero and introduced a broader package of monetary easing. As market-based indicators of longer-term inflation expectations subsequently declined, the ECB increasingly emphasised the risk that low inflation could become embedded in wage- and price-setting (European Central Bank 2014, Draghi 2014*a*,*b*).

In January 2015, the ECB expanded its asset purchase programme to include public-sector securities. The decision was explicitly motivated by the risk of a too prolonged period of low inflation, at a time when indicators of both actual and expected inflation had fallen close to historical lows (European Central Bank 2015). The deterioration of the 5y5y BoR identified in our estimates is therefore consistent with the policy concern underlying these measures: low inflation was increasingly being priced not only as a near-term consequence of weak demand, but as a risk extending into the true long-term horizon.

The BoR remained negative at all three horizons during most of the second half of the decade. The continued weakness of the 5y5y measure indicates that the challenge was not fully resolved by the initial recovery in realised inflation or economic activity. In 2019,

renewed declines in inflation expectations and persistently subdued inflation prompted a further package of measures, including an additional reduction in the deposit facility rate and the restart of net asset purchases (European Central Bank 2019). The term structure of the BoR thus mirrors the changing nature of the policy problem faced by the ECB: initially a cyclical deterioration concentrated at short horizons, and later a persistent below-target inflation risk extending to the nominal anchor.

4.3.2 The COVID-19 shock

The outbreak of COVID-19 generated a second and more abrupt deterioration in the inflation outlook. The pandemic struck after several years in which below-benchmark inflation risks were already entrenched, and the BoR fell sharply at all three horizons during the first half of 2020. The decline was particularly notable in the 5y5y measure, which reached its lowest value in the sample. This suggests that the initial pandemic shock was not priced solely as a temporary disruption, but also intensified concerns about the persistence of the pre-existing low-inflation environment.

The composition of the RNDs nevertheless differed markedly across horizons. The probability of outright deflation rose to almost 70% at the two-year horizon and to around 36% at the five-year horizon. At the true long-term horizon, however, the probability of negative inflation remained below 7%. The deterioration in the 5y5y outlook instead reflected a marked increase in the probability of low positive inflation, with the probability assigned to the 0–1.5% range exceeding 70% at its peak.

The term structure therefore distinguishes two related but different risks. At short and medium horizons, markets priced a substantial possibility of outright deflation. Between five and ten years ahead, the dominant concern was not deflation itself, but the persistence of inflation rates materially below the price-stability benchmark. Once again, the BoR captures this broader deterioration more fully than a measure based exclusively on the probability of negative inflation.

From the autumn of 2020, the BoR began to recover across horizons, although it remained negative. This improvement coincided with additional monetary-policy measures, the announcement and subsequent deployment of vaccines, and a gradual reassessment of the economic outlook. However, option prices continued to assign greater probability mass to below-benchmark than to above-benchmark inflation outcomes.

4.3.3 The post-pandemic inflation surge

The post-pandemic inflation surge produced a rapid reversal in the direction of risks to price stability. From 2021 onwards, the reopening of the economy, supply disruptions and rising energy prices coincided with a pronounced rightward shift in the inflation RNDs. The repricing was initially concentrated at the two-year horizon but subsequently extended to the five-year and 5y5y distributions. By early 2022, the BoR had moved into positive territory at all three horizons for the first time in over a decade.

The magnitude of the shift varied substantially across the term structure. In spring 2022, the BoR reached approximately 0.98 at the two-year horizon and 0.88 at the five-year horizon, compared with a considerably more moderate maximum of around 0.42 for the 5y5y distribution reached only in May 2023.

The sharp rise in the short- and medium-term measures indicates that option prices assigned an overwhelming share of probability mass to inflation above 2% over the nearer horizons. The more contained response of the 5y5y BoR shows that markets also revised

long-term risks upwards, but did not price an equally pronounced entrenchment of high inflation between five and ten years ahead.

After their peaks in 2022-23, the BoR measures declined progressively towards more balanced levels. This moderation coincided with the tightening of monetary policy, the easing of some supply constraints and declining energy prices. The correction was again strongest at short and medium horizons. By contrast, the more moderate initial increase in the 5y5y BoR and its subsequent decline suggest that the post-pandemic inflation shock generated only a limited and comparatively temporary deterioration in true long-term inflation risks. Short-term BoR rose sharply again at the end of February 2026 reflecting a revision of inflation risks due to start of the military conflict in the Middle East.

Taken together, these episodes show that neither a single maturity nor a single tail probability provides a complete assessment of risks to price stability. During the 2010s, relatively contained deflation probabilities coexisted with severe downside risks arising from the concentration of the RND at low positive inflation rates. During the pandemic, deflation risks rose sharply at shorter horizons, whereas the true long-term deterioration was driven primarily by the probability of persistently low inflation. Conversely, the post-pandemic surge generated extremely pronounced upside risks at short and medium horizons but a more contained response in the 5y5y distribution. The joint analysis of the whole RND and its term structure thus provides information both about the direction of inflation risks and about their perceived persistence.

5 The pricing of inflation risks

To better understand the evolution of risks to price stability, this section investigates which macroeconomic and financial conditions are associated with the pricing of inflation risks in the euro area, and how their relevance varies across maturities.

5.1 Empirical methodology

Our goal here is to find evidence on which specific macroeconomic and financial variables help understand the dynamics of risks to price stability documented above. Research work on the links between inflation risks and macroeconomic variables have followed two very different approaches. Some contributions have employed event-study approaches to explore the reactions of (risk-neutral) inflation probabilities to macroeconomic news (inflation, oil prices, etc) at high frequencies across different horizons (e.g., Kitsul & Wright 2013, Galati et al. 2018). In contrast, other contributions have explored the explanatory power of a low-dimensional set of macroeconomic and financial variables for (short-term) tail risk (physical) probabilities estimated via a parsimonious structural macromodel (e.g., López-Salido & Loria 2024) . Despite differences in empirical methodologies, variables and horizons, existing results point to an important variation over time in the drivers of inflation risks.

Against that background, we take an agnostic approach to explore the drivers of risks to price stability. Financial markets *de facto* operate in a data-rich environment, with a large number of indicators potentially informing the assessment of the inflation outlook (e.g., Stock & Watson 1999, Bernanke & Boivin 2003). Our empirical approach therefore considers 36 directly observable variables grouped into nine broad categories: money aggregates and credit, foreign exchange and commodities, prices and costs, economic

activity, macroeconomic risks, confidence indicators, money and bond markets, stock markets, and international factors. Broadly speaking, our macroeconomic variables extend those in (López-Salido & Loria 2024) while the additional set of financial variables should also help explain shifts in market sentiment and risk compensation. Our aim is to provide evidence that the dynamics of risks to price stability can be related to macroeconomic and financial conditions at frequencies that can help monetary policy analysis. Table 1 reports the full set of candidate variables and the transformations applied to them.

For each horizon, we model the balance of risks as

$$\text{BoR}_t = a\ \text{BoR}_{t-1} + \mathbf{b}'\mathbf{X}_t + \varepsilon_t, \tag{18}$$

where BoR_t denotes the balance of risks at the corresponding horizon and $\mathbf{X}_t$ is the vector of potential explanatory variables. The lagged dependent variable captures persistence in the pricing of inflation risks, while the contemporaneous regressors capture their association with current macroeconomic and financial conditions. Given the substantial heterogeneity among the potential explanatory candidates, all variables, including the BoR, are standardised over the whole sample period.

With 36 potential explanatory variables, model uncertainty is substantial: even keeping the autoregressive term in all specifications, the candidate set contains 2^{36} possible combinations of regressors.[8]. To address the variable selection challenge, our empirical application relies on the Stochastic Search Variable Selection (SSVS) approach of George & McCulloch (1993).

The model Equation (18) can be written in compact form as $Y_t = \mathbf{Z}_t'\boldsymbol{\beta} + \varepsilon_t$, with $\varepsilon_t \sim N(0, \sigma^2)$. Under standard assumptions for the SSVS framework,[9] the conditional posterior distributions of the $\boldsymbol{\beta}$ coefficients and other model parameters are available in closed form and can be sampled using a Gibbs algorithm. To recover them, we run 500,000 Gibbs iterations and discard the first 100,000 as burn-in.

The hyperparameters are chosen symmetrically across regressors and within the ranges suggested by George & McCulloch (1993):[10]

$$\nu_\gamma = 6, \qquad \delta_\gamma = 0.7, \qquad p_j = 0.5, \qquad \tau_j = 0.01, \qquad c_j = \frac{3}{\tau_j} \quad \forall j.$$

The posterior estimates of the corresponding $\boldsymbol{\beta}$ coefficients for each variable can then be used to calculate the historical contribution of each variable to the BoR at each horizon according to Equation (18). The contributions by types of factors are then just the result of adding the contribution of all the individual variables belonging to each factor category.

[8] An exhaustive comparison of 2^{36} specifications would be computationally prohibitive using conventional model-selection criteria such as AIC or BIC. At the same time, while statistical factors could be extracted from the whole list of variables or each individual group, that will lower the transparency of the effects which is a main goal of our exercise.

[9] The prior for coefficient β_j is $\beta_j \mid \gamma_j \sim (1-\gamma_j)N(0, \tau_j^2) + \gamma_j N(0, c_j^2\tau_j^2)$, where $\gamma_j \in \{0, 1\}$ is a latent inclusion indicator. Specifically, when $\gamma_j = 0$, the small prior variance τ_j^2 strongly shrinks β_j towards zero. When $\gamma_j = 1$, the larger variance $c_j^2\tau_j^2$ allows the coefficient to take economically meaningful values. The prior inclusion probability is $P(\gamma_j = 1) = p_j$, and, assuming prior independence across inclusion indicators, $P(\boldsymbol{\gamma}) = \prod_j p_j^{\gamma_j}(1-p_j)^{1-\gamma_j}$. The hierarchical prior is completed by assuming $\sigma^2 \mid \boldsymbol{\gamma} \sim IG\left(\frac{\nu_\gamma}{2}, \frac{\delta_\gamma}{2}\right)$. For the general derivation of the SSVS posterior, see George & McCulloch (1993). A related exposition under similar assumptions is provided by Koop et al. (2007).

[10] Robustness exercises using alternative prior hyperparameters within the ranges suggested by George & McCulloch (1993) leave the main results unchanged.

Table 1: Potential explanatory factors for inflation risks.

No.	Variable	Country	Transformation
Money aggregates and credit			
1	M1, index of notional stocks, seasonally adjusted	Euro area	yoy
2	M3, index of notional stocks, seasonally adjusted	Euro area	yoy
3	Loans to non-financial corporations	Euro area	yoy
Foreign exchange and commodities			
4	ECB nominal effective exchange rate (NEER)	Euro area	yoy
5	World market prices of raw materials excluding energy (HWWA index)	Euro area	yoy
6	Brent crude oil price in U.S. dollars	Euro area	yoy
Prices and costs			
7	HICP, overall index, seasonally adjusted	Euro area	yoy
8	HICP excluding energy and unprocessed food, seasonally adjusted	Euro area	yoy
9	HICP volatility	Euro area	24m rolling s.d.
10	HICP excluding energy and unprocessed food, volatility	Euro area	24m rolling s.d.
11	PPI, total industry excluding construction, seasonally adjusted	Euro area	yoy
12	ECB index of negotiated wages, three-month moving average	Euro area	yoy, 3mma
Economic activity			
13	Industrial production excluding construction, working-day and seasonally adjusted	Euro area	yoy
14	Unemployment rate	Euro area	12m change
Macroeconomic risks			
15	Probability of low inflation one year ahead, ECB's SPF	Euro area	Probability
16	Probability of high inflation one year ahead, ECB's SPF	Euro area	Probability
17	Probability of recession one year ahead, ECB's SPF	Euro area	Probability
18	Probability of high growth one year ahead, ECB's SPF	Euro area	Probability
19	Probability of low inflation over the long term, ECB's SPF	Euro area	Probability
20	Probability of high inflation over the long term, ECB's SPF	Euro area	Probability
Confidence indicators			
21	Consumer Confidence Indicator, seasonally adjusted	Euro area	Level
22	Composite PMI	Euro area	Level
Money and bond markets			
23	Euribor–OIS spread	Euro area	Spread
24	Euro area yield-curve slope (10y–3m)	Euro area	Spread
25	10-year U.S. Treasury–German Bund yield differential	Euro area–U.S.	Spread
26	U.S. yield-curve slope (10y–3m)	U.S.	Spread
27	Bund implied volatility	Euro area	Level
28	Credit spread	Euro area	Level
Stock markets			
29	EURO STOXX 50 index	Euro area	12m return
30	S&P 500 index	U.S.	12m return
31	VIX	U.S.	Level
32	VDAX–VSTOXX	Euro area	Level
International factors			
33	U.S. consumer price index, all urban consumers, seasonally adjusted	U.S.	yoy
34	U.S. industrial production, seasonally adjusted	U.S.	yoy
35	U.S. non-farm payrolls	U.S.	12m change
36	IMF world output gap	World	Level

Note: The table reports the 36 potential explanatory variables for risk to price stability, grouped into nine different categories. All variables are stationary or transformed to achieve stationarity. "yoy" denotes the year-on-year growth rate; "24m rolling s.d." denotes the standard deviation calculated over a 24-month rolling window; "12m change" denotes the change relative to twelve months earlier; and "12m return" denotes the twelve-month percentage return. Sources: ECB, Eurostat, Bloomberg, Refinitiv, Hamburgisches Welt-Wirtschafts-Archiv (HWWA), ECB Survey of Professional Forecasters (SPF), IMF, and authors' calculations. Data are obtained from the respective institutions' websites or from subscription-based data services.

5.2 Inflation risks and macro-financial factors: overview

Our results point to substantial differences in the pricing of perceived inflation risks at short-to-medium (over the next five years) and longer-term horizons (five-to-ten years ahead).[11] To illustrate, Figures 3 and 4 display the contributions of the nine types of factors used in our analysis (see Table 1 above).

In terms of broad factor categories, as expected, Prices and Costs indicators comprising key variables to understand and forecast inflation dynamics are priced in at all horizons. Short-to-medium term inflation risks however also reflect factors more closely associated to "cyclical" economic conditions, like FX and Commodities and Confidence Indicators (see Figure 3). This may be somewhat more surprising, since their impact should be expected to fade away over the medium-term. However, exploiting the advantages of the methodology described in previous Sections, a decomposition of the 5y RND into inflation risks over a shorter-term horizon (next two years, 2y RND), and medium-term risks (between two and five years ahead, 3y2y RND) confirms their importance at the latter horizon as well. In addition, Macroeconomic Risks, Money and Bond Market and Stock Market indicators also provide important information for inflation risks at medium-term horizons, albeit they play a lower quantitative role. In contrast, our analysis suggests that Money Aggregates and Credit, Economic Activity and International factors do not appear to be relevant for pricing inflation risks.

Long-term inflation risks (BoR five-to-ten years ahead) also reflect developments in Price and Cost indicators, but financial factors (mainly Money and Bond market factors) play an important explanatory role, possibly shaping not only expected inflation but also inflation risk premia at long-term horizons (see Figure 4). Macroeconomic Risks are also important pricing factors to understand the fluctuations in long-term inflation risks. In contrast, FX and Commodities and, particularly, Confidence Indicators, which, as discussed above, explain a substantial part of the variation in perceived risks over the next five years, do not seem to be relevant at longer horizons. Besides, Monetary Aggregates and Credit, Economic Activity, Stock Market and International Macroeconomic factors do not seem to affect inflation risks materially at any horizon either.

[11] To focus discussion, our analysis here concentrated on the difference between short-to-medium (over the next five years) and longer term risks (five-to-ten years ahead) which seems to be particularly relevant for the euro area. Our forward RND estimation methodology can also be applied to other segments of the term structure of inflation risks, for example the 3y2y RND. We will make use of insights from such a decomposition of inflation risks over the next five years when helpful to better understand them. Detailed results are available upon request.

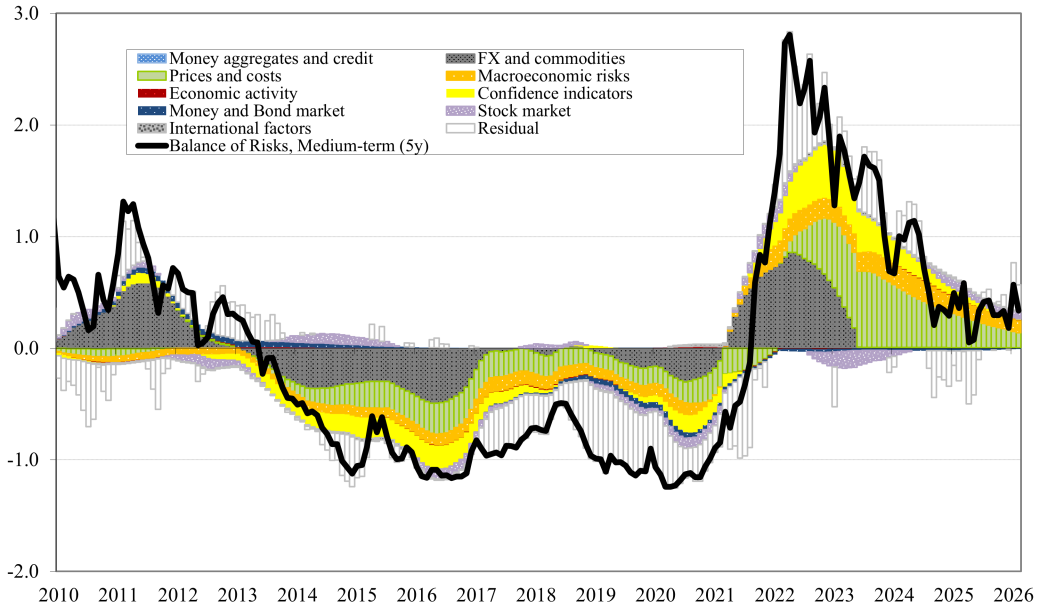


Figure 3: **Drivers of inflation risks in RND at medium-term horizon (5y RND): contributions by type of factor.**
Note: The chart depicts the decomposition of the Balance of Risks in the short-to-medium-term inflation RND (5y, reflecting risks over the next 5 years) as the sum of the dynamic contribution of the potential explanatory variables/type of factors (see Table 1 for details) and the residual term. All variables are stationarised and the historical values of the endogenous variable should therefore be interpreted as their departure from a baseline or reference path. The sum of all contributions, including the residual, therefore returns the historical values of the endogenous variable in deviation from the baseline. The contribution is based on the posterior mean from the Stochastic Search Variable Selection (SSVS) approach, see Section 5.1 and George & McCulloch (1993) for additional details. For presentational purposes, the individual variable contributions are aggregated into the 9 groups of factors to which they belong as detailed in Table 1. Sample: 2010M1-2026M2.

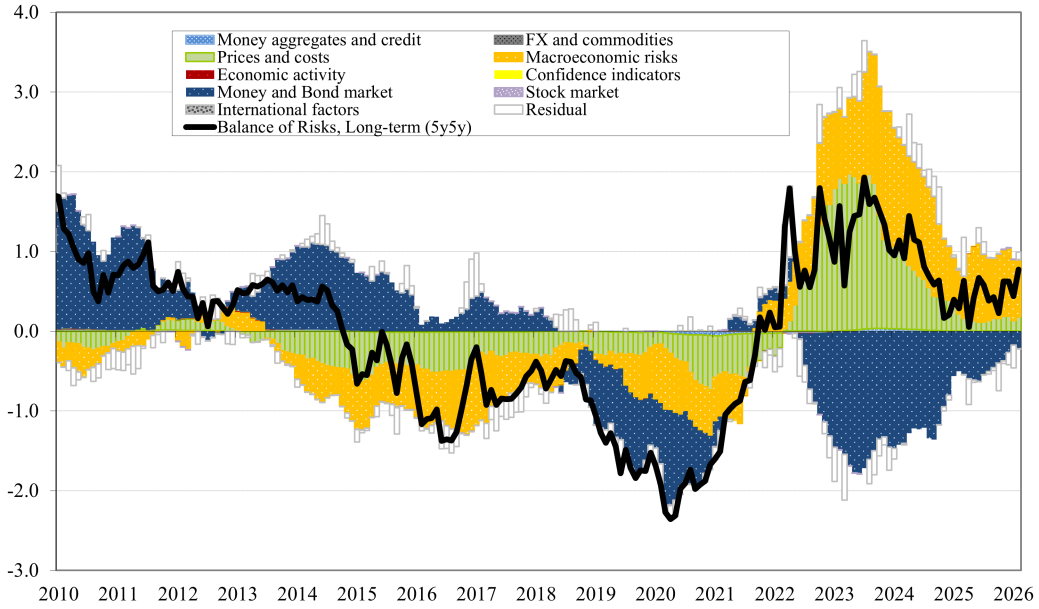


Figure 4: **Drivers of inflation risks at long-term horizons (5y5y RND): contributions by type of factor.**
Note: The chart depicts the decomposition of the Balance of Risks in the long-term inflation RND (5y5y, reflecting risks between 5 and 10 years ahead) as the sum of the dynamic contribution of 36 potential explanatory variables grouped by type of factors (see Table 1 for details) and the residual term. All variables are standardised and the historical values of the endogenous variable should therefore be interpreted as their departure from a baseline or reference path. The sum of all contributions, including the residual, therefore returns the historical values of the endogenous variable in deviation from the baseline. The contribution is based on the posterior mean from the Stochastic Search Variable Selection (SSVS) approach, see Section 5.1 and George & McCulloch (1993) for additional details. For presentational purposes, the individual variable contributions are aggregated into the 9 groups of factors to which they belong as detailed in Table 1. Sample: 2010M1-2026M2.

5.3 Historical analysis of inflation risks

We next exploit the specific variable contributions to further examine how specific macroeconomic and financial conditions are associated with the evolution of inflation risks over time. This allows us to move from the broad factor categories discussed above to a historical assessment of the individual indicators underlying changes in the BoR.

The estimated specifications provide a close in-sample fit at both horizons. The R^2 is around 95% for both the five-year and 5y5y BoR, and the fitted series closely track the observed balance of risks over the sample (Figures 5 and 6). We therefore use the estimated contributions of the individual variables to examine the main episodes of downside and upside risks to price stability in the following subsections.

5.3.1 Drivers of downside risks to price stability: the 2010s and the COVID-19 outbreak

The unprecedented nature of the Global Financial Crisis (GFC) raised uncertainty about future inflation, with evolving concerns about both potential deflation and high inflation reflecting the unprecedented macro-financial conditions, as well as the uncertainty about the impact of the unconventional monetary policy measures launched at the time by the ECB (and other major central banks). Indeed, euro area-wide inflation readings shifted from a brief spell of negative readings in the second half of 2009 to temporary highs at around 3% by the second half of 2011. In addition, the sharp rebound in oil and commodity prices supported by strong demand from China and Emerging Economies over 2009-2010, and to a lesser extent the recovery of stock markets also raised inflation risks.

However, as the European debt crisis deepened, low inflation concerns gradually intensified, as reflected in the BoR at short-to-medium horizons becoming negative since July 2011. Our analysis shows that the downward repricing of inflation risks can be largely explained by three key factors (see Figure 5). Firstly, low actual inflation, both headline and core, particularly in some euro area countries, contributed to the decline not only in inflation expectations but also in the overall balance of risks to price stability. Secondly, PMI fluctuations, capturing deteriorating business confidence, also played a crucial role. Moreover, both persistently below-target inflation and subdued business confidence seem to have been a drag on inflation risks over the low inflation period for most of the 2010s. Finally, the declines in commodity prices (oil and other raw materials) as a result of the slowing Chinese industrial demand and global oversupply, particularly over 2014-16, reinforced downside risks, and counteracted positive influences by other factors, like the long-term bond yield differential between U.S. treasuries and German bunds, particularly over the ZLB period in U.S. rates. In addition, the protracted period of subdued expectations for euro area growth (over the short-term) and of high inflation (over the long-term) contributed to keep risks to price stability on the downside for the rest of the 2010s.

While short-to-medium risks to price stability were in a protracted downward trend since 2010, long-term inflation risks were less affected by the conjunctural situation in the euro area around the European debt crisis period. Since 2014, however, the downward revision of the BoR into negative territory at longer horizons (five-to-ten years ahead) has been driven by two key economic developments. First, the persistently low headline inflation: euro area-wide inflation fell below the 2% level in February 2013, and, apart from a brief spell barely above that level between May and October 2018, it remained below-target until July 2021. It is therefore not surprising that headline inflation developments

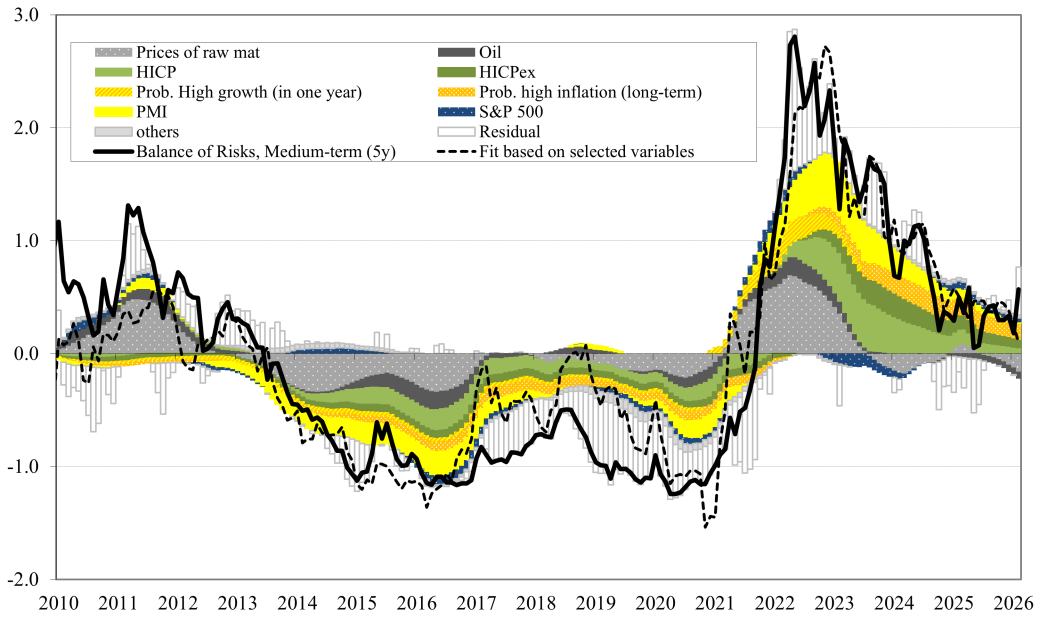


Figure 5: **Drivers of inflation risks at medium-term horizon (5y RND): contributions from specific variables.**
Note: The chart depicts the decomposition of the Balance of Risks in the short-to-medium-term inflation RND (5y, reflecting risks over the next 5 years) as the sum of the dynamic contribution of the potential explanatory variables (see Table 1 for details) and the residual term. All variables are standardised and the historical values of the endogenous variable should therefore be interpreted as their departure from a baseline or reference path. The sum of all contributions, including the residual, therefore returns the historical values of the endogenous variable in deviation from the baseline. The contribution is based on the posterior mean from Stochastic Search Variable Selection (SSVS) approach, see Section 5.1 and George & McCulloch (1993) for additional details. Sample: 2010M1-2026M2.

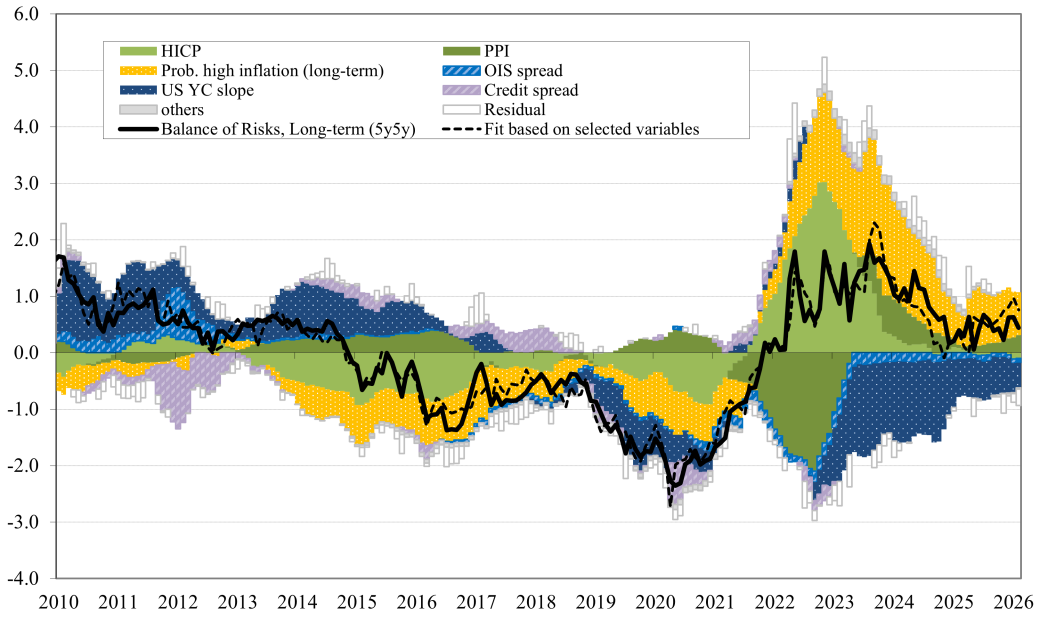


Figure 6: **Drivers of inflation risks at long-term horizons (5y5y): contributions from specific variables.**
Note: The chart depicts the decomposition of the Balance of Risks in the long-term (5y5y) inflation RND estimated in this paper as the sum of the dynamic contribution of the potential explanatory variables (see Table 1 for details) and the residual term. All variables are stationarised and the historical values of the endogenous variable should therefore be interpreted as their departure from a baseline or reference path. The sum of all contributions, including the residual, therefore returns the historical values of the endogenous variable in deviation from the baseline. The contribution is based on the posterior mean from the Stochastic Search Variable Selection (SSVS) approach, see Section 5.1 and George & McCulloch (1993) for additional details. Sample: 2010M1-2026M2.

exerted a negative contribution on inflation risks for most of the 2010s. Interestingly, the pricing of long-term inflation risks also takes into account influences in price pressures at earlier stages in the production chain (as captured by developments in the PPI, see Figure 6). The pricing of several indicators of inflationary pressures suggests a thorough monitoring of price pressures in the options market, and their need to correctly identify persistent pressures that will influence inflation over long horizons. The fact that the protracted period of below-target headline inflation dynamics counteracted PPI influences nonetheless suggests that market participants truly expected a rather subdued inflation trend to persist (Garcia et al. 2026). Second, and also pointing to an important revision to inflation risks, the downward revision in the BoR over the long term also reflects lower survey-based expected probability of high inflation at the long-term horizon in the ECB's SPF.

Those two economic indicators largely explain the protracted period of downside risks to price stability in the euro area for most of the 2010s. In addition, positive influences on inflation risks from other monetary and financial conditions that had contributed to partially counteract those downside risks in the first part of the 2010s, gradually disappeared from 2015. First and foremost, the removal of the highly accommodative U.S. monetary policy (as captured by the slope of the U.S. yield curve) exerted a significant positive

contribution to euro area long-term inflation risks in the first part of the decade (2010-16) when the Federal Reserve engaged in significant and protracted monetary policy stimulus, but negatively since 2016 when that stimulus was gradually withdrawn.

Widening Euribor-OIS spreads reflecting interbank credit and funding stress as a result of the evolving European debt crisis also contributed to raise long-term inflation risks in the early 2010s. This suggests that there were significant concerns about the potential consequences of the expansionary monetary policy response to counteract that crisis (Draghi 2012). However, the decisive introduction of the Outright Monetary Transactions (OMT) programme as well as Long-Term Refinancing Operations (LTROs) successfully averted that tail risk but the inflationary threat did not materialise and the positive contribution from the money-market spreads declined significantly from 2013. That interpretation is also consistent with the identified role for credit spreads, whose widening in the context of the European debt crisis coincides with sizable negative contributions to the BoR. The launching of Asset Purchase Programmes since 2014 (particularly those for corporate assets, CBPP3, CSPP, ABSPP)[12] however supported the compression of credit spreads, and led to their positive contributions to long-term inflation risks over the rest of the decade.

The outbreak of COVID-19 in early 2020 and the associated decline in economic activity as a result of the lockdowns in many euro area countries further exacerbated deflation fears, with macroeconomic and financial conditions severely impacting risks to price stability at short-to-medium and longer-term horizons through the same channels highlighted above. At short-to-medium term horizons, inflation readings, which quickly turned into negative territory in 2020, severely-hit business confidence and a decline in oil and commodity prices following the slowdown in global economic activity boosted deflationary pressures to unprecedented levels. Downside risks to price stability at long horizons reflected the deterioration in actual inflation readings, declining expectations of high inflation, the widening of the OIS and credit spreads and the relatively flat U.S. yield curve despite the Federal Reserve's aggressive cuts in 2020.

5.3.2 Factors behind the return of high inflation and upside inflation risks: 2022-26

Risks to price stability shifted dramatically along the recovery from the COVID-19 pandemic: the BoR rose sharply at all horizons and turned positive, pointing to upside risks to price stability from late 2021. As economic activity rebounded, the sustained rise in inflation from 2021, peaking above 10% in late 2022, was the main driver of the strong upward revision in short-to-medium term inflation risks. Along with the more sustained re-openings and recovery in economic activity with the roll-out of the vaccines from 2021, increasing business confidence turned to exert a positive contribution to the rise in inflation risks, which peaked towards the end of 2022. In addition, the rebound in oil and commodity prices since 2021, was further supported by geopolitical conflicts affecting energy prices in the euro area and contributed to the rise in inflation risks.

Interestingly, short-term macroeconomic risks as reflected in macroeconomic surveys, which were not quantitatively important for most of the sample, also played a role in the post-pandemic period. Specifically, the probability of high growth one year ahead was also an important driver of the inflation risks since late 2020, most likely reflecting the perception that demand pressures could outpace the recovery in production after the

[12] A description of the ECB's asset purchase programmes is available on the ECB website.

lockdowns and the disruptions in global supply chains.

Overall, however, the analysis of the term structure of inflation risks suggests that there was a growing perception of inflationary pressures becoming more persistent over time. In addition to the clear shift towards upside risks to price stability (BoR into positive territory at all horizons), the probability assigned to high inflation outcomes (e.g. above 4%) two years ahead peaked at 19% in early 2023, and a bit later (over the summer of 2023) for longer horizons, both over the next five years (at 16%) and between five-and-ten years ahead (at 13.9%). Moreover, their decline has been very gradual thereafter, still averaging 9.7% and 8.6% respectively over the twelve months following their peaks, despite the fact that by September 2024 headline inflation was again below the 2% level. The perceived probability of high inflation over the long-term, which had remained rather subdued and tended to diminish inflation risks for most of the sample, became an important driver of the BoR since 2023, underscoring the importance of the anchoring of inflation expectations.

The sharp rise in the long-term BoR was also driven by higher inflation (both headline and core) and higher expected probability of high inflation (probability above 3% at the long-term horizon in the ECB's SPF peaked at almost 30% in late 2022). As those pressures attenuated, the BoR eased gradually, also influenced by the significant monetary policy tightening on both sides of the Atlantic (and worldwide). Indeed, there is an important negative contribution from the U.S. yield curve slope since 2022, reflecting the sharp tightening of monetary policy to curtail inflation risks.

Overall, the relationships unveiled in this section show that the pricing of inflation risks in option markets can not only be linked to observable factors but that their evolution tends to reflect closely the evolution of the economic environment. While our analysis is mostly empirical, we believe it sets sound foundations for a more structural analysis of the inflation options market.

6 Concluding remarks

This paper has analysed the information content of the term structure of risks implied by inflation options traded in the euro area market between 2009-2026. We propose a robust methodology based on two main elements. First, zero-coupon RNDs are estimated using a non-parametric (model-free) approach. Second, to evaluate forward inflation RNDs for horizons of interest (e.g., the long-term five-year forward inflation RND in five years that is currently non-traded in the options market) marginal RNDs are coupled via a Student's t-copula, fitted to inflation swap rate data. Employing this novel, robust methodology which relies on a very limited number of assumptions, we can obtain information about the pricing of inflation risks at short, medium and long-term horizons based on traded inflation derivatives, which offers valuable insights for monetary policy and investors.

Focusing on the euro area inflation experience, we have shown that monitoring the whole term structure of inflation expectations and associated risks could offer important insights into how persistent shocks to inflation and inflation expectations are perceived to be. Moreover, we have provided empirical evidence that different macroeconomic and financial variables help explain risks to price stability at different horizons, both for short-to-medium and longer-term ones. In the first half of 2020 the risk-neutral probability of outright deflation reached almost 70% two years ahead and 36% at five years, but stayed below 7% at 5y5y; by spring 2022 the wedge had reopened with the opposite sign. Whether a shock is perceived as persistent is visible in the term structure of risks and in nothing else we observe.

Our findings carry a direct implication for how inflation risks are monitored in practice. The five-year horizon, widely used as the benchmark for assessing inflation risks (e.g. ECB 2018, IMF 2023), averages over exactly the divergence that matters: it is the horizon at which the near-term repricing has largely faded but the long-horizon anchor has not yet been reached. Reading inflation risks off that horizon alone can therefore understate how well anchored expectations are during a disinflation, and overstate it during a surge.

Further analysis on the risk premia and the inflation density under the "physical" measure embodied in the RNDs could provide additional insights on inflation expectations formation[13] and should be an important topic for future research. The evidence on the macroeconomic and financial drivers of inflation RNDs documented in this paper provides strong support regarding their information content at different horizons, and should help develop multivariate yet parsimonious models for that line of research. More immediately, the framework requires only instruments that are already liquid in most developed inflation markets, so the term structure of inflation risks documented here for the euro area can be constructed wherever a central bank needs to know whether a shock to the inflation outlook is being priced as temporary or as permanent.

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[13] Applying the methodology for estimating inflation RNDs developed in this paper, for instance, Garcia et al. (2026) reveal important differences in the pass-through from inflation and short-term expectations on medium and long-term inflation expectations and risks between the euro area and the U.S.

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Appendix

A Pricing of zero-coupon inflation-linked instruments

This appendix provides additional details on the risk-neutral valuation of the zero-coupon inflation-linked instruments introduced in Section 2.2. Standard references on inflation-linked markets and their valuation include Deacon et al. (2004), Canty & Heider (2012), and Brigo & Mercurio (2007).

Let $I(t)$ denote the relevant consumer price index and let the annualised inflation rate between dates t and T, denoted by $\pi(t,T)$, be defined by

$$\frac{I(T)}{I(t)} = (1+\pi(t,T))^{T-t}.$$

Let $r_n(t)$ denote the nominal short rate and let $P_n(t,T)$ be the time-t price of a default-free nominal zero-coupon bond paying one unit of currency at maturity T. Under the risk-neutral measure $\mathbb{Q}$,

$$P_n(t,T) = \mathbb{E}_{\mathbb{Q}}\left[\exp\left\{-\int_t^T r_n(s)\,ds\right\}\middle|\,\mathcal{F}_t\right],$$

where $\mathcal{F}_t$ denotes the information available at time t.

A.1 Zero-coupon inflation-indexed swaps

A zero-coupon inflation-indexed swap (ZCIIS) initiated at time zero and maturing at time T exchanges the realised cumulative inflation payoff

$$N\left[(1+\pi(0,T))^T - 1\right]$$

against the fixed payoff

$$N\left[(1+k)^T - 1\right],$$

where N is the notional and k is the fixed annualised swap rate.

From the perspective of the fixed-rate payer, the value of the contract at time $t \le T$ is

$$\text{ZCIIS}(t,T,k) = \mathbb{E}_{\mathbb{Q}}\left[\exp\left\{-\int_t^T r_n(s)\,ds\right\} N\left((1+\pi(0,T))^T - (1+k)^T\right)\middle|\,\mathcal{F}_t\right]. \quad \text{(A.1)}$$

At inception, the fair swap rate is chosen such that the value of the contract is zero:

$$\text{ZCIIS}(0,T,k) = 0.$$

Using the nominal zero-coupon bond as numeraire and applying the standard change-of-numeraire result (Jamshidian 1989, Brigo & Mercurio 2007), equation (A.1) can be written under the T-forward measure $\mathbb{Q}^T$ as

$$\text{ZCIIS}(0,T,k) = NP_n(0,T)\left\{\mathbb{E}_{\mathbb{Q}^T}\left[(1+\pi(0,T))^T\right] - (1+k)^T\right\}.$$

Imposing the zero-value condition therefore gives

$$(1+k)^T = \mathbb{E}_{\mathbb{Q}^T}\left[(1+\pi(0,T))^T\right]. \quad \text{(A.2)}$$

Using

$$(1+\pi(0,T))^T = \frac{I(T)}{I(0)},$$

the fair-swap condition may equivalently be expressed as

$$(1+k)^T = \frac{1}{I(0)}\mathbb{E}_{\mathbb{Q}^T}\left[I(T)\right].$$

The forward CPI associated with maturity T is defined as

$$\mathcal{I}_T(t) := \mathbb{E}_{\mathbb{Q}^T}\left[I(T) \mid \mathcal{F}_t\right].$$

Consequently, at inception,

$$\mathcal{I}_T(0) = I(0)(1+k)^T.$$

When it is useful to emphasise both maturity and observation date, we denote the market-observed swap rate by $k_T(t)$.

A.2 Zero-coupon inflation caps and floors

A zero-coupon inflation cap with strike k and maturity T pays

$$\left[(1+\pi(0,T))^T - (1+k)^T\right]_+$$

at maturity. Its value at time $t \le T$ under the risk-neutral measure is

$$C(t;T,k) = \mathbb{E}_{\mathbb{Q}}\left[\exp\left\{-\int_t^T r_n(s)\,ds\right\}\left[(1+\pi(0,T))^T - (1+k)^T\right]_+ \middle| \mathcal{F}_t\right].$$

Changing from $\mathbb{Q}$ to the T-forward measure yields

$$C(t;T,k) = P_n(t,T)\mathbb{E}_{\mathbb{Q}^T}\left[\left[(1+\pi(0,T))^T - (1+k)^T\right]_+ \middle| \mathcal{F}_t\right]. \tag{A.3}$$

Similarly, a zero-coupon inflation floor pays

$$\left[(1+k)^T - (1+\pi(0,T))^T\right]_+,$$

and its time-t value is

$$P(t;T,k) = P_n(t,T)\mathbb{E}_{\mathbb{Q}^T}\left[\left[(1+k)^T - (1+\pi(0,T))^T\right]_+ \middle| \mathcal{F}_t\right]. \tag{A.4}$$

The corresponding put–call parity relation is

$$C(t;T,k) - P(t;T,k) = P_n(t,T)\left\{\mathbb{E}_{\mathbb{Q}^T}\left[(1+\pi(0,T))^T \middle| \mathcal{F}_t\right] - (1+k)^T\right\}.$$

At inception, using Equation (A.2), this becomes

$$C(0;T,k) - P(0;T,k) = P_n(0,T)\left[(1+k_T(0))^T - (1+k)^T\right]. \tag{A.5}$$

Equations (A.3) and (A.4) show that zero-coupon inflation caps and floors can be viewed, respectively, as European call and put options written on

$$S(T) = \frac{I(T)}{I(0)} = (1 + \pi(0,T))^T ,$$

with strike

$$K = (1+k)^T.$$

This option representation allows us to apply the Breeden–Litzenberger result (Breeden & Litzenberger 1978), according to which the second derivative of the call-price function with respect to the strike recovers, up to the nominal discount factor, the risk-neutral density of the underlying cumulative inflation ratio $I(T)/I(0)$. Section 3.1 develops its empirical implementation.

B Lee's growth condition

This appendix shows that the transformation and extrapolation of implied volatility used in Section 3.1 satisfy the asymptotic tail-growth restriction of Lee (2004).

Let $\sigma_{\text{imp}}(K)$ denote the implied volatility of an option with strike K, maturity T, and forward price F. A necessary asymptotic condition for an arbitrage-free implied-volatility smile is that implied variance does not grow faster than linearly in absolute log-moneyness. In particular, the asymptotic growth of total implied variance satisfies

$$\limsup_{|\log(K/F)|\to\infty} \frac{\sigma^2_{\text{imp}}(K)T}{|\log(K/F)|} \leq 2.$$

with the corresponding condition applying to both the right and left tails. This implies

$$\sigma^2_{\text{imp}}(K) = \mathcal{O}\left(|\log(K/F)|\right), \qquad K \to 0 \quad \text{or} \quad K \to \infty. \tag{B.1}$$

We transform implied volatility according to

$$\widetilde{\sigma}(K) = \exp\left\{\sigma^2_{\text{imp}}(K)\right\}.$$

Equivalently,

$$\sigma^2_{\text{imp}}(K) = \log \widetilde{\sigma}(K).$$

Suppose that the positive extrapolation of $\widetilde{\sigma}(K)$ has polynomial growth or slower. For the right tail, this means that, for sufficiently large K, there exist constants $a > 0$, $b > 0$, and $m \geq 0$ such that

$$\widetilde{\sigma}(K) \leq a + bK^m.$$

It follows that

$$\sigma^2_{\text{imp}}(K) = \log \widetilde{\sigma}(K) \leq \log\left(a + bK^m\right) = \mathcal{O}(\log K), \qquad K \to \infty.$$

An analogous argument applies to the left tail when the extrapolation grows no faster than a polynomial in K^{-1}. Hence,

$$\sigma^2_{\text{imp}}(K) = \mathcal{O}\left(|\log(K/F)|\right), \qquad K \to 0 \quad \text{or} \quad K \to \infty,$$

as required by Equation (B.1).

In our implementation, a natural cubic smoothing spline is fitted to $\widetilde{\sigma}(K)$ over the observed strike interval and extrapolated linearly using its endpoint slopes. Since linear growth is a special case of polynomial growth, the extrapolated implied variance satisfies Lee's tail-growth restriction, provided that the extrapolation preserves the domain of the inverse transformation, $g(K) \geq 1$.

C Analytical recovery of the inflation RND

This appendix derives the analytical expression used to recover the risk-neutral density from the fitted implied-volatility smile.

Let $C(t;T,K)$ denote the price of a European call option with strike K and maturity T, written on an underlying asset with forward price F. Under the Black representation,

$$C(t;T,K) = P_n(t,T)\left[F\Phi(d_1) - K\Phi(d_2)\right],$$

where

$$d_{1,2} = \frac{\log(F/K) \pm \frac{1}{2}\sigma^2_{\text{imp}}(K)(T-t)}{\sigma_{\text{imp}}(K)\sqrt{T-t}},$$

and $\Phi(\cdot)$ denotes the standard normal cumulative distribution function.

Allowing implied volatility to depend on the strike, the Breeden–Litzenberger relationship (Breeden & Litzenberger 1978) gives

$$q_S^T(T,K) = \frac{1}{P_n(t,T)}\frac{\partial^2 C(t;T,K)}{\partial K^2}.$$

Differentiating the Black price analytically yields

$$q_S^T(T,K) = \phi(d_2)\Bigg[\frac{1}{K\sigma_{\text{imp}}(K)\sqrt{T-t}} + \frac{2d_1}{\sigma_{\text{imp}}(K)}\frac{\partial\sigma_{\text{imp}}(K)}{\partial K} + \frac{Kd_1d_2\sqrt{T-t}}{\sigma_{\text{imp}}(K)}\left(\frac{\partial\sigma_{\text{imp}}(K)}{\partial K}\right)^2 + K\sqrt{T-t}\frac{\partial^2\sigma_{\text{imp}}(K)}{\partial K^2}\Bigg],$$

where $\phi(\cdot)$ is the standard normal density.

The fitted natural cubic smoothing spline, denoted by $g(K)$, is applied to the transformed implied volatility

$$\widetilde{\sigma}(K) = \exp\left\{\sigma^2_{\text{imp}}(K)\right\},$$

so that

$$g(K) = \widetilde{\sigma}(K).$$

The motivation for this transformation is discussed in Appendix B.

Define

$$L(K) := \log g(K).$$

It follows that

$$\sigma_{\text{imp}}(K) = \sqrt{L(K)},$$

with first derivative

$$\frac{\partial\sigma_{\text{imp}}(K)}{\partial K} = \frac{g'(K)}{2g(K)\sqrt{L(K)}},$$

and second derivative

$$\frac{\partial^2\sigma_{\text{imp}}(K)}{\partial K^2} = \frac{g''(K)}{2g(K)\sqrt{L(K)}} - \frac{[g'(K)]^2}{2g(K)^2\sqrt{L(K)}} - \frac{[g'(K)]^2}{4g(K)^2[L(K)]^{3/2}}.$$

Let $\tau = T - t$. Substituting these expressions into the analytical Breeden–Litzenberger density gives

$$q_S^T(T,K) = \phi(d_2)\left[\frac{1}{K\sqrt{L(K)}\sqrt{\tau}} + \frac{d_1 g'(K)}{g(K)L(K)} + \frac{K d_1 d_2 \sqrt{\tau}\,[g'(K)]^2}{4g(K)^2[L(K)]^{3/2}} + K\sqrt{\tau}\left\{\frac{g''(K)}{2g(K)\sqrt{L(K)}} - \frac{[g'(K)]^2}{2g(K)^2\sqrt{L(K)}} - \frac{[g'(K)]^2}{4g(K)^2[L(K)]^{3/2}}\right\}\right],$$

where

$$d_{1,2} = \frac{\log(F/K) \pm \frac{1}{2}L(K)\tau}{\sqrt{L(K)\tau}}.$$

The expression above recovers the risk-neutral density of the option underlying, $S(T)$, directly from the fitted spline and its first two analytical derivatives.

For zero-coupon inflation options, the underlying is the cumulative inflation index ratio,

$$S(T) = \frac{I(T)}{I(0)} = (1 + \pi(0,T))^T.$$

The density recovered above is therefore the risk-neutral density of cumulative inflation. To express the distribution in terms of the annualised inflation rate, note that

$$\pi(0,T) = S(T)^{1/T} - 1.$$

Let x denote a possible realisation of the annualised inflation rate. Since

$$S = (1+x)^T,$$

the corresponding Jacobian is

$$\frac{dS}{dx} = T(1+x)^{T-1}.$$

Hence, the density of annualised inflation is

$$q_\pi^T(T,x) = T(1+x)^{T-1} q_S^T\left(T,(1+x)^T\right). \tag{C.1}$$

At each date and maturity, the fitted spline and its analytical derivatives are first used to recover q_S^T. Equation (C.1) then transforms this density into the RND of the annualised inflation rate. This analytical procedure avoids the numerical instability associated with applying finite differences twice to an interpolated option-price curve.

D Implementation of the spot RND estimator

This appendix illustrates the spot RND estimation procedure described in Section 3.1. We use five-year euro area zero-coupon inflation cap and floor quotes observed on 24 March 2014.

Table 2: Five-year zero-coupon inflation cap and floor prices on 24 March 2014

Strike	−0.020	−0.015	−0.010	−0.005	0.000	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050	0.060
Floor price	1.520	2.900	4.447	12.86	28.00	71.15	149.9	306.7	504.7	735.6	986.7	—	—	—	—	—
Cap price	—	—	—	—	—	—	251.0	144.5	83.83	48.77	29.84	18.95	—	8.420	5.840	1.405

Note: The table reports prices of five-year euro area zero-coupon inflation caps and floors for a notional amount of 10,000. A dash indicates that no quote is available.

D.1 Option quotes and call-equivalent prices

Table 2 reports the available cap and floor prices across annualised inflation strikes. Prices correspond to a notional amount of 10,000.

To combine the information contained in caps and floors, floor prices are converted into call-equivalent prices using put–call parity. At inception,

$$C(0;T,k) = P(0;T,k) + P_n(0,T)\left[(1+k_{\text{swap}})^T - (1+k)^T\right],$$

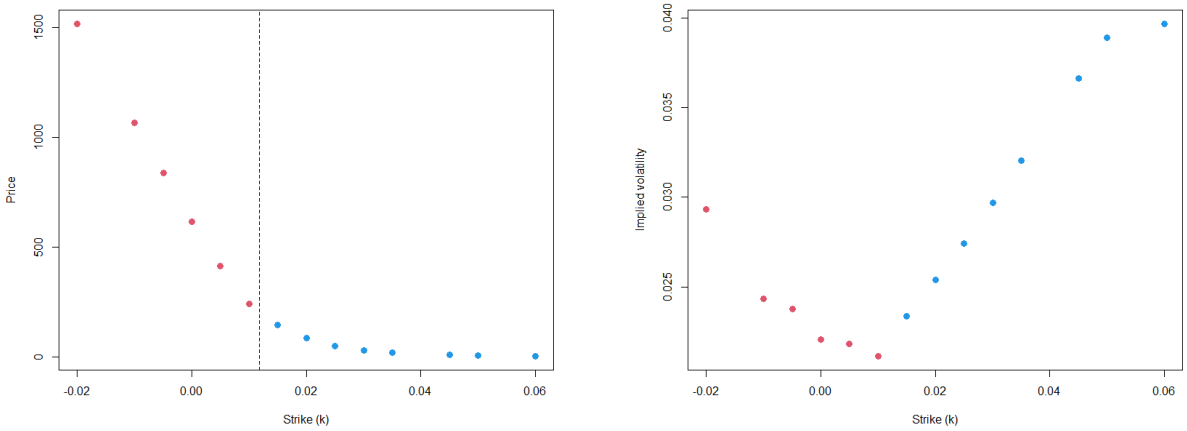
where k_{swap} is the prevailing zero-coupon inflation swap rate. The discount factor $P_n(0,T)$ is obtained from the OIS curve.

Whenever both a cap and a floor are quoted at the same strike, we retain the out-of-the-money contract, following standard practice (e.g., Aït-Sahalia & Lo 1998, Gimeno & Ibáñez 2018). Floors are therefore used for strikes below the prevailing swap rate, whereas caps are used for strikes above it.

The annualised quoted strike k_i is transformed into the strike on the cumulative inflation index ratio:

$$K_i = (1+k_i)^T, \qquad i = 1,\ldots,n.$$

Figure 7 displays the observed and call-equivalent cap prices and the corresponding Black implied volatilities.



(a) Cap and call-equivalent prices

(b) Black implied volatilities

Figure 7: Five-year zero-coupon inflation option prices and implied volatilities on 24 March 2014

Note: The left panel shows observed cap prices and cap prices derived from floor quotes using put–call parity. The right panel shows their corresponding Black implied volatilities. The vertical dashed line indicates the prevailing five-year inflation swap rate.

D.2 Spline estimation and density recovery

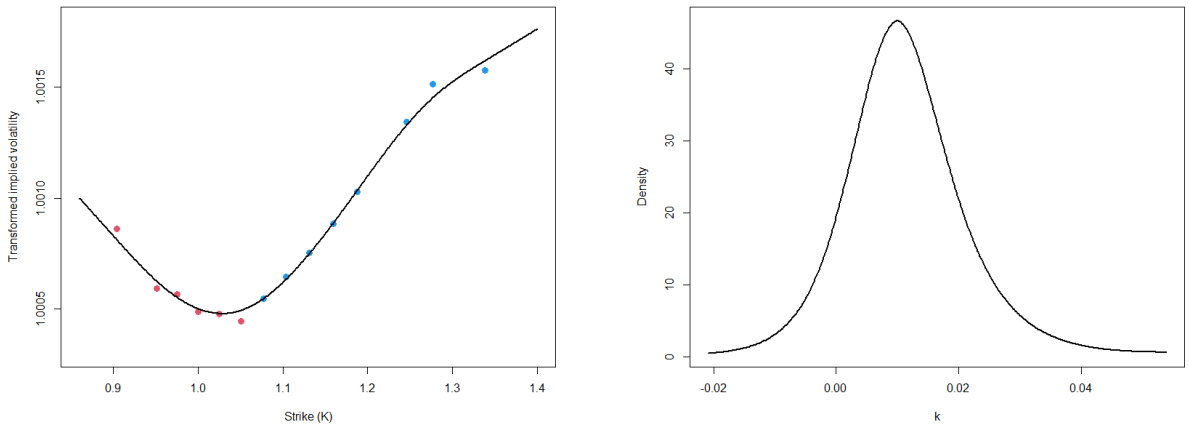
The Black implied volatilities are transformed according to

$$\widetilde{\sigma}_i = \exp\left\{\sigma^2_{\text{imp}}(K_i)\right\}.$$

A natural cubic smoothing spline is then fitted to the pairs $(K_i, \widetilde{\sigma}_i)$. In the benchmark implementation, the smoothing parameter is set to $\alpha = 0.5$.

The fitted spline is extrapolated linearly beyond the lowest and highest observed strikes using the corresponding endpoint slopes. The RND is then computed using the analytical expressions in Appendix C.

Figure 8 shows the fitted transformed implied-volatility curve and the corresponding RND of the annualised inflation rate over the following five years.



(a) Fitted transformed implied-volatility smile

(b) RND of $\pi(0, 5)$

Figure 8: Fitted volatility smile and five-year inflation RND on 24 March 2014

Note: The left panel reports the transformed implied volatilities and the fitted natural cubic smoothing spline. The right panel reports the corresponding risk-neutral density of the annualised inflation rate over the following five years.

The example illustrates how information from caps and floors across strikes is combined to obtain a smooth density without imposing a parametric distribution on future inflation.

E Robustness of the RND estimation

This appendix examines the robustness of the RND estimation methodology. We first evaluate the accuracy of the interpolation and extrapolation procedure using option prices generated from a model with a known implied density. Subsequent subsections assess the choice of copula and the rolling-window length used to construct forward RNDs. These exercises assess the estimation methodology rather than the empirical results reported in Sections 4 and 5, and they are conducted on the sample through May 2023.

E.1 Accuracy of the spot RND estimator

We evaluate the spot RND estimator using synthetic zero-coupon inflation option prices generated from the SABR-type inflation model of Mercurio & Moreni (2009). The advan-

tage of this exercise is that the true option-implied density is known and can therefore be compared directly with the density recovered by our non-parametric procedure.

For simplicity, we consider a one-period model:

$$\begin{aligned} dI(t) &= I(t)V(t)\,dW(t), \\ dV(t) &= \nu V(t)\,dZ(t), \\ dW(t)dZ(t) &= \rho_{WZ}\,dt, \end{aligned}$$

with initial conditions

$$I(0) = 1 + k, \qquad V(0) = v_0.$$

The parameter vector is

$$\theta = (\nu, v_0, \rho_{WZ})'.$$

For each simulation, the parameters are drawn independently from

$$\begin{aligned} \nu &\sim U(0.001, 0.4), \\ v_0 &\sim U(0.005, 0.02), \\ \rho_{WZ} &\sim U(-1, 1). \end{aligned}$$

Conditional on the parameter draw and an assumed inflation swap rate, we generate synthetic zero-coupon cap prices at the same strikes observed in the empirical dataset. We then apply the smoothing-spline estimator described in Section 3.1 and compare the resulting probabilities with those implied by the data-generating model.

We consider both tail and interior probability ranges. The tail probabilities are

$$\Pr\left(\pi(0,1) < -0.02\right) \quad \text{and} \quad \Pr\left(\pi(0,1) > 0.06\right),$$

while the interior ranges correspond to economically relevant inflation intervals used in the empirical analysis. For each swap-rate value, the exercise is repeated over 10^4 parameter draws. Table 3 reports the mean absolute percentage errors for the two tail probabilities.

Table 3: Accuracy of the estimated tail probabilities

Swap rate k	$\Pr(\pi(0,1) < -0.02)$	$\Pr(\pi(0,1) > 0.06)$
	Mean absolute percentage error (%)	
0.000	0.77	2.30
0.005	1.07	2.47
0.010	1.58	2.42
0.015	2.45	2.34
0.020	4.12	2.08
0.025	6.34	1.64

Table 4 reports the corresponding errors for probability intervals within the observed strike range. The errors are very small for probabilities within the observed strike range. As expected, extrapolation produces larger errors in the tails, particularly in the lower tail when the swap rate is relatively high. Even in those cases, however, the estimated probabilities remain reasonably close to their model-implied counterparts. The exercise therefore supports the accuracy of the interpolation and extrapolation procedure used in the empirical analysis.

Table 4: Accuracy of the estimated interior probabilities

Swap rate k	$\Pr(0 < \pi(0,1) < 0.015)$	$\Pr(0.015 < \pi(0,1) < 0.02)$	$\Pr(0.02 < \pi(0,1) < 0.03)$
	Mean absolute percentage error (%)		
0.000	0.04	0.19	0.22
0.005	0.03	0.11	0.15
0.010	0.03	0.06	0.08
0.015	0.02	0.05	0.05
0.020	0.04	0.05	0.04
0.025	0.13	0.05	0.05

E.2 Alternative copula specifications

The Student's t-copula is the benchmark specification used to construct forward inflation RNDs. We compare its empirical performance with two alternatives: the Gaussian copula and the grouped Student's t-copula.

The Gaussian copula is a limiting case of the Student's t-copula as $\nu \to \infty$ and therefore rules out tail dependence. The grouped Student's t-copula is more flexible, allowing the two components to have different degrees-of-freedom parameters. This additional flexibility may improve the in-sample fit but also increases the dimensionality and numerical complexity of the estimation.

For each rolling estimation window, we compute the maximised log-likelihood under the three copula specifications. To facilitate comparison across dates, the likelihood of each model is expressed relative to the highest likelihood attained within the same window. We then average these relative likelihoods over the full sample. Table 5 reports the results.

Table 5: Comparison of copula likelihoods

Copula	Gaussian	Student's t	Grouped Student's t
Mean percentage of best likelihood	0.933	0.989	0.916

The Student's t-copula provides a better fit than the Gaussian copula, supporting the inclusion of tail dependence. The grouped Student's t-copula performs less well over the full sample, partly because its optimisation fails to converge in some rolling windows.

To distinguish numerical instability from goodness of fit, we repeat the comparison using only dates on which the grouped Student's t-copula converges successfully. The results are reported in Table 6.

Table 6: Comparison of copula likelihoods conditional on successful grouped-t estimation

Copula	Gaussian	Student's t	Grouped Student's t
Mean percentage of best likelihood	0.933	0.989	1.000

Conditional on successful convergence, the grouped Student's t-copula provides the best fit, but its improvement over the standard Student's t-copula is small. The standard Student's t-copula therefore offers the best balance between flexibility, fit, and numerical stability, and is used as the benchmark specification.

E.3 Sensitivity to the rolling-window length

The benchmark copula parameters are estimated using a rolling window of 100 trading days. This choice balances two competing considerations. A shorter window allows the dependence structure to adjust more rapidly to new market conditions but produces less precise parameter estimates. A longer window provides more stable estimates but may react more slowly to changes in the pricing of inflation risks.

We assess the sensitivity of the estimated forward RNDs to alternative rolling windows of 45 and 150 trading days. For each specification, we reconstruct the 5y5y RND and compare three measures used in the empirical analysis: the probability of deflation, the probability of inflation above 3%, and the balance of risks around the 2% price-stability benchmark. Figure 9 compares the probability of inflation below zero across the three window lengths. Figure 10 reports the corresponding probability of inflation exceeding 3%.

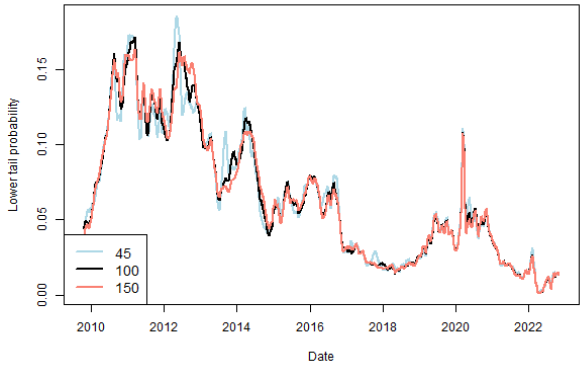


Figure 9: Probability of inflation below zero under alternative rolling windows
Note: The figure reports the probability assigned by the 5y5y RND to annualised inflation below zero when the Student's t-copula is estimated using rolling windows of 45, 100, and 150 trading days. Sample: 2009M10–2023M5.

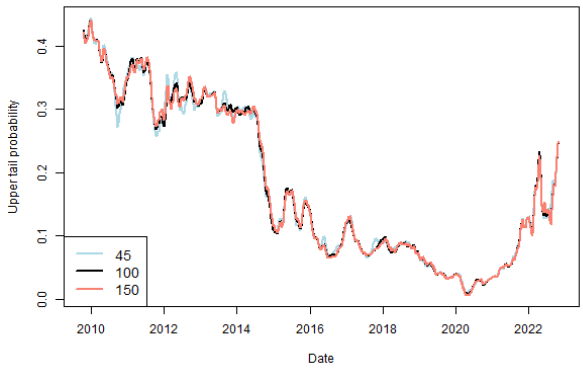


Figure 10: Probability of inflation above 3% under alternative rolling windows
Note: The figure reports the probability assigned by the 5y5y RND to annualised inflation above 3% when the Student's t-copula is estimated using rolling windows of 45, 100, and 150 trading days. Sample: 2009M10–2023M5.

Finally, Figure 11 compares the balance of risks across the three specifications. The differences across the three specifications are small. The main historical movements in

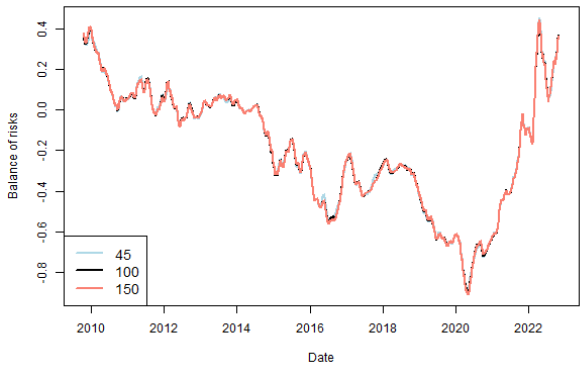


Figure 11: Balance of risks under alternative rolling windows

Note: The figure reports the balance of risks derived from the 5y5y RND when the Student's t-copula is estimated using rolling windows of 45, 100, and 150 trading days. Sample: 2009M10–2023M5.

the left tail, the upper tail, and the balance of risks are preserved independently of the window length. The benchmark results are therefore not driven by the choice of a 100-day rolling window.

F Student's t-copula and forward-density construction

This appendix provides additional details on the Student's t-copula and the construction of forward inflation risk-neutral densities described in Section 3.2.

F.1 The Student's t-copula

Let $t_\nu(\cdot)$ denote the cumulative distribution function of a univariate standard Student's t distribution with ν degrees of freedom, and let $t_{\rho,\nu}(\cdot,\cdot)$ denote the cumulative distribution function of a standardised bivariate Student's t distribution with correlation parameter ρ and ν degrees of freedom (Joe 2015). The bivariate Student's t-copula is defined as

$$C_{\rho,\nu}(u_1,u_2) = t_{\rho,\nu}\left(t_\nu^{-1}(u_1), t_\nu^{-1}(u_2)\right), \tag{F.1}$$

for $(u_1,u_2) \in (0,1)^2$. Its density is

$$c_{\rho,\nu}(u_1,u_2) = \frac{f_{\rho,\nu}\left(t_\nu^{-1}(u_1), t_\nu^{-1}(u_2)\right)}{f_\nu\left(t_\nu^{-1}(u_1)\right) f_\nu\left(t_\nu^{-1}(u_2)\right)}, \tag{F.2}$$

where $f_{\rho,\nu}$ denotes the density of the standardised bivariate Student's t distribution and f_ν the density of the corresponding univariate distribution.

For a bivariate copula C with uniform margins (U_1,U_2), the coefficients of upper and lower tail dependence are defined as the limiting conditional probabilities:

$$\lambda_U = \lim_{u\to 1^-} \Pr\left(U_2 > u \mid U_1 > u\right), \qquad \lambda_L = \lim_{u\to 0^+} \Pr\left(U_2 \le u \mid U_1 \le u\right), \tag{F.3}$$

whenever these limits exist. They measure the tendency of the two variables to attain extreme values jointly. They are properties of the copula alone, i.e. they do

not depend on the marginal distributions. In our setting, $U_1 = F_{T_1}(I(T_1)/I(0))$ and $U_2 = F_{T_2}(I(T_2)/I(0))$. Hence, λ_U is the limiting $\mathbb{Q}$-probability that cumulative inflation to T_2 is extreme given that cumulative inflation to T_1 is equally extreme.

For the Gaussian copula, $\lambda_U = \lambda_L = 0$ for every correlation coefficient $|\rho| < 1$, i.e. extreme outcomes are asymptotically independent however strong the correlation. The Student's t-copula nests the Gaussian as $\nu \to \infty$, but for finite ν the two coefficients are equal by radial symmetry (Joe 2015):

$$\lambda := \lambda_U = \lambda_L = 2t_{\nu+1}\left(-\sqrt{\frac{(\nu+1)(1-\rho)}{1+\rho}}\right). \tag{F.4}$$

Hence, lower values of ν imply stronger joint tail dependence, conditional on ρ.

F.2 Joint distribution of cumulative inflation

Let

$$Y_1 = \frac{I(T_1)}{I(0)}, \qquad Y_2 = \frac{I(T_2)}{I(0)}, \qquad 0 < T_1 < T_2. \tag{F.5}$$

Let f_{T_1} and f_{T_2} denote their marginal densities and F_{T_1} and F_{T_2} their corresponding cumulative distribution functions. The marginal distributions are recovered from zero-coupon inflation option prices using the procedure described in Section 3.1.

The joint density is constructed through the Student's t-copula:

$$f_{T_1,T_2}(y_1, y_2) = c_{\rho,\nu}\left(F_{T_1}(y_1), F_{T_2}(y_2)\right) f_{T_1}(y_1) f_{T_2}(y_2). \tag{F.6}$$

Following Gimeno & Ibáñez (2018), we assume deterministic interest rates. Under this assumption, the T_1- and T_2-forward measures coincide with a common risk-neutral measure $\mathbb{Q}$. Consequently, the marginal RNDs recovered at the two maturities can be combined under the same pricing measure.

F.3 Density of the cumulative forward inflation ratio

The cumulative inflation ratio over the interval $[T_1, T_2]$ is

$$X = \frac{I(T_2)}{I(T_1)} = \frac{Y_2}{Y_1}. \tag{F.7}$$

To derive its density, consider the transformation

$$(y_1, y_2) \longmapsto (y_1, x), \qquad x = \frac{y_2}{y_1}. \tag{F.8}$$

The inverse transformation is

$$y_2 = x y_1. \tag{F.9}$$

The absolute value of the Jacobian determinant is

$$\left|\frac{\partial(y_1, y_2)}{\partial(y_1, x)}\right| = |y_1|. \tag{F.10}$$

Therefore, the density of $X = Y_2/Y_1$ is

$$q_{T_1,T_2}(x) = \int_{\mathbb{R}} |y_1| f_{T_1,T_2}(y_1, x y_1)\, dy_1, \tag{F.11}$$

following the standard result for the density of a quotient of two random variables (Curtiss 1941).

Since price indices are strictly positive, $Y_1 > 0$ and $Y_2 > 0$. Hence, Equation (F.11) can be written more simply as

$$q_{T_1,T_2}(x) = \int_0^\infty y_1 f_{T_1,T_2}(y_1, xy_1)\, dy_1, \qquad x > 0. \tag{F.12}$$

Substituting Equation (F.6) gives

$$q_{T_1,T_2}(x) = \int_0^\infty y_1 c_{\rho,\nu}\left(F_{T_1}(y_1), F_{T_2}(xy_1)\right) \times f_{T_1}(y_1) f_{T_2}(xy_1)\, dy_1. \tag{F.13}$$

F.4 Density of annualised forward inflation

The annualised inflation rate over the interval $[T_1, T_2]$ satisfies

$$X = \left(1 + \pi(T_1, T_2)\right)^{T_2 - T_1}. \tag{F.14}$$

Let

$$Z = \pi(T_1, T_2). \tag{F.15}$$

Then

$$X = (1 + Z)^{T_2 - T_1}, \tag{F.16}$$

and

$$\frac{dX}{dZ} = (T_2 - T_1)(1 + Z)^{T_2 - T_1 - 1}. \tag{F.17}$$

The risk-neutral density of the annualised forward inflation rate is therefore

$$q_\pi^{T_1,T_2}(z) = (T_2 - T_1)(1 + z)^{T_2 - T_1 - 1} q_{T_1,T_2}\left((1 + z)^{T_2 - T_1}\right), \tag{F.18}$$

for $z > -1$.

For the 5y5y inflation rate, $T_1 = 5$ and $T_2 = 10$, so

$$q_\pi^{5,10}(z) = 5(1 + z)^4 q_{5,10}\left((1 + z)^5\right). \tag{F.19}$$

F.5 Estimation of the copula parameters

Let $k_T(t)$ denote the observed T-year zero-coupon inflation swap rate at date t, and define

$$K_T(t) = \left(1 + k_T(t)\right)^T. \tag{F.20}$$

Under deterministic interest rates, the fair-swap condition implies that $K_T(t)$ captures the risk-neutral valuation of cumulative inflation at maturity T, up to the scaling by the current price index discussed in Appendix A.

We use daily changes

$$\Delta K_T(t) = K_T(t) - K_T(t - 1) \tag{F.21}$$

to estimate the dependence between maturities. At each observation date t, the copula parameters (ρ_t, ν_t) are estimated using the rolling sample

$$\mathcal{D}_t = \left\{\left(\Delta K_{T_1}(s), \Delta K_{T_2}(s)\right)\right\}_{s=t-n+1}^{t}. \tag{F.22}$$

The parameters are obtained by maximum likelihood. Let $\widehat{G}_{1,t}$ and $\widehat{G}_{2,t}$ denote the empirical marginal distribution functions of the two series within the rolling window. The pseudo-observations are

$$u_{1,s} = \widehat{G}_{1,t}\left(\Delta K_{T_1}(s)\right), \qquad u_{2,s} = \widehat{G}_{2,t}\left(\Delta K_{T_2}(s)\right). \tag{F.23}$$

In implementation, the empirical ranks are scaled to lie strictly inside the unit interval:

$$u_{j,s} = \frac{R_{j,s}}{n+1}, \qquad j = 1, 2, \tag{F.24}$$

where $R_{j,s}$ is the rank of observation s in marginal series j.

The log-likelihood function is

$$\ell_t(\rho, \nu) = \sum_{s=t-n+1}^{t} \log c_{\rho,\nu}\left(u_{1,s}, u_{2,s}\right). \tag{F.25}$$

The parameter estimates are

$$(\widehat{\rho}_t, \widehat{\nu}_t) = \arg\max_{\rho,\nu} \ell_t(\rho, \nu), \tag{F.26}$$

subject to

$$-1 < \rho < 1, \qquad \nu > 0. \tag{F.27}$$

The benchmark specification uses a rolling window of 100 trading days. The window ends at the estimation date, ensuring that only information available at that date is used. Alternative window lengths and alternative copula specifications are examined in Appendix E.

F.6 Estimated tail dependence

In Appendix E.2 we have shown that the Student's t-copula is preferred to the Gaussian. In this subsection we report what that preference amounts to in economic terms by quantifying disagreement between the two copulas.

As we discussed in Appendix F.1, a Gaussian copula rules out tail dependence, $\lambda_U = \lambda_L = 0$ for every value of the correlation parameter ρ, consequently extreme outcomes at the two maturities are asymptotically independent regardless of how strongly their distributions co-move. In contrast, the Student's t-copula can model tail dependence.

Figure 12 reports time evolution of the tail-dependence coefficient λ. We see that tail dependence is substantial throughout the sample: the median is 0.62, the tenth and ninetieth percentiles are 0.26 and 0.77, and the coefficient exceeds 0.30 on 88% of trading days and 0.50 on 69%. Therefore, in the euro area inflation option market, an extreme five-year cumulative inflation outcome is, in the limit, accompanied by an equally extreme ten-year outcome with probability roughly 0.6. Recall that the Gaussian copula specifications used in the literature of (e.g., Gimeno & Ibáñez 2018, Hilscher et al. 2022) rule out this probability by construction.

Two features are worth noting. First, the level is stable. The yearly medians lie between 0.25 and 0.79, and exceed 0.38 in every year of the sample except 2014. Second, the series falls close to zero on a small number of dates. These are windows in which the estimated degrees-of-freedom parameter runs towards its Gaussian limit, and they are informative rather than anomalous since the specification is free to return the Gaussian case and occasionally does. Thus, the tail dependence reported here is a property of the data rather than an artefact of the functional form.

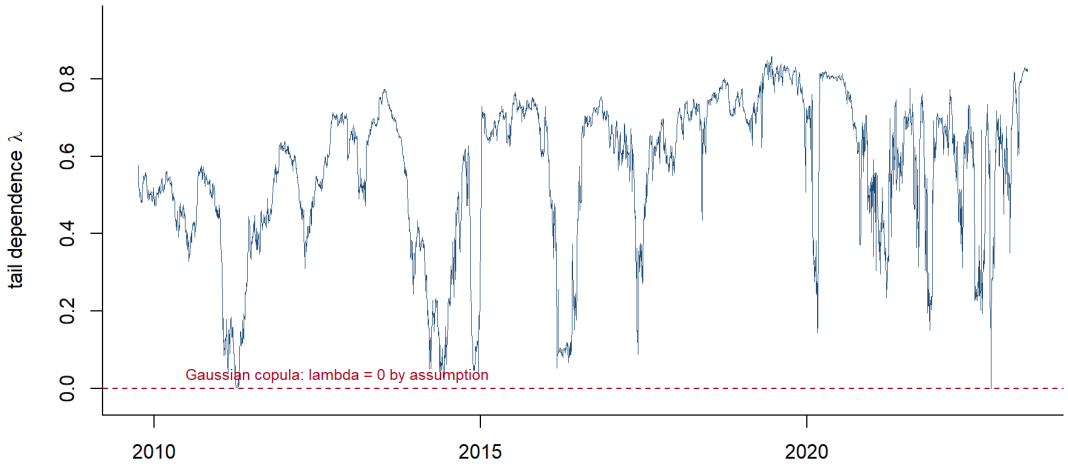


Figure 12: Estimated tail dependence between five- and ten-year cumulative inflation

Note: The figure reports the coefficient of tail dependence λ from (F.4) implied by the fitted Student's t-copula parameters evaluated at each estimation date. The dashed line marks the value imposed by a Gaussian copula, which is always zero by construction.

Acknowledgements

We are particularly grateful to Matteo Ciccarelli, Carlos Montes-Galdon, Christian Speck, Sebastian Werner, and seminar participants at the 54th Money, Macro and Finance conference, the 20th World Congress of the IEA, the ECB and the Bank of Spain for useful comments and suggestions.
The views expressed are those of the authors and do not necessarily represent those of the ECB and Banco de España, their Executive Boards or their management.

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PDF ISBN 978-92-899-8032-6 ISSN 1725-2806 doi:10.2866/2393703 QB-01-26-232-EN-N